LAGADHA

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Abstract: In this article, the ancient Indian astronomer Lagadha is presented as a multifaceted personality. His traits and achievements are explained to those with a minimal knowledge of Sanskrit. The aspects touched upon are:

1. Lagadha is an ancient author.
2. Lagadha is a preserver of tradition.
3. Lagadha is a devotee.
4. Lagadha is an astronomer.
5. Lagadha is a ritualist.
6. Lagadha is a poet.
7. Lagadha is a player of words with mild humour.
8. Lagadha is a cryptic writer.
9. Lagadha is wellversed in arithmetical calculations.
10. Lagadha is a leader and populariser of astronomy.

We illustrate these with four instances for each.

1.Lagadha is the author of a small book called Vedaangajyotisham. This is the oldest available fullfledged text of Indian astronomy. It is a book of Sanskrit verses. The whole book does not exceed 7 or 8 pages. It is believed that Lagadha lived approximately three and a half thousand years ago. Though we may not be able to fix his exact age, we put forward four arguments to prove the antiquity of this work. These are based respectively on an astronomical clue, a linguistic clue, a structural clue and a content-based clue. All these four are internal evidences. We quote the actual passages in this book where these clues are available. Of these four, the first one has been elaborately explained by the historians of Mathematics.

1.1.There is a passage in Lagadha's book that leads us to the conclusion that at his time the asterism sravishtha was in winter equinox. The sloka runs as follows.

praapadyete shravishtaadau suryaacandramasaavudak.
Saarpaardhe dakshinaarkastu maaghashraavanayostadaa

If it is so, then by a fairly routine calculation, assigning 72 years for each degree (and therefore approximately 960 years for each asterism or constellation) one can arrive at the fact that Lagadha's period could be around 1350 B.C. It should however be remarked that in the astronomical cycle of events, the same would have been the situation at 27270 B.C. (and at 53190 B.C. etc.). But the historians rule out the possibility of so great an antiquity, for other reasons, that are not impeccable. We will now go along with them. After all, it is along the same lines, a rough date is assigned to the (still earlier) vedas also, by these scholars.

1.2.The language experts have their own way of estimating the age of ancient texts. They look at the vocabulary; they see the style; they take the grammatical aspects into account; they compare these with other works of known periods. If we want to employ these methods for ascertainig the approximate age of Lagadha's work, we have to make the following observations. The language of Lagadha is occasionally (more often than not?) not understandable.

Here is what an expert writes: "It is mostly filled with unintelligible rubbish, and leaves us in the lurch as regards valuable information" These are the words of Whitney, whom many historians of Indian texts respect very much. Even a scholar who disagreed with this opinion of Whitney and questioned, "if unintelligible, how does he decide it is rubbish?", had lamented "of obscure terms and apothegmatic language" in Lagadha's book. The more unintelligible its Sanskrit, the more ancient it is likely to be. Even where the meanings of all words are understood, there are occasions where the passage makes no sense. Here are two instances:

mrudu pancadashaashtame.

dyu heyam parva cetpaade.

What does softness have to do in this context? What sense does it make to omit a day if parva is in the quarter? Is the book so old that the text has been mutilated? Can we make intelligent guesses what the correct version would have been? Thankfully, some scholars like T.S.Kuppanna Sastry (hereafter abbreviated as TSK) have already done this job. It is for us to continue it. Now it suffices to say that such a predicament is one of the indicators of antiquity.

1.3.At the end of the book (Vedaangajyotisha) one finds a line (added later) which a novice may discard as nonsensical. But the traditionalists, who can compare this with what they learn at similar contexts of vedic recensions can decode what it is.

pancasamvatsaram prapadyete kaaryaa: kalaa dasha ca yaa:

parva savitaa vishuvam sapta.

Comparing this with a line like "ishe drugmha bhuvanam ashtaavigmshati:" (at the end of the first chapter of Yajurveda) one readily makes out that this is a sequence of words occurring in the beginnings of different subsections or at regular intervals. But the point that we are driving to is different. This practice of having such lines at the end (that are really not a part of the book) is solely intended for the convenience of those who get the whole book by heart. This practice must have been there only for those books that were in use before the practice of writing had a firm root. We don't find such a thing in the Sanskrit books authored in the last 2500 years.

1.4.We look at the contents of the book, particularly the technical terms there. When most of the later astronomy books mention such terms as mesha raashi, vrushabharaashi, etc.(twelve parts of the zodiac), bhaanu vaara, somavaara, etc.(days of the week), and hora etc, we find that no such terms or their equivalents are found in Vedaangajyotisham. The units of time seem to be more or less (if not exactly) same as the ones mentioned in vedas. This adds strength to our contention that this book was composed at a time not far from the time of revelation of vedas.

All these clues pave the way for concluding that the period of Lagadha is before 1150 B.C. We do not know how much before.

2.Lagadha is a preserver of tradition. He values the vedic tradition very much. We prove this point through four instances.

2.1.He does not claim originality, though there are some items that are not available to us from anywhere else except from Lagadha and his followers. But Lagadha explicitly mentions that these are handed over to him through generations. Cf. the sentences like

kaalajnaanam pracaxate. shaastrajnai: smrutam.

It may not be just out of his modesty. He may be actually correct when he says that these facts are known in India from a remote past. Lagadha, more than in creating new knowledge, takes pride in saying that he preserves the knowledge of his ancestors.

2.2.He takes care to assert that his views are acceptable to his fellow-brahmins. He asserts that his findings are in conformity with the traditional knowledge. He is not a radical or a revolutionary thinker. This is what he writes in the beginning of his book:

vipraanaam sammatam loke

In another version of his book, the phrase is slightly modified as:

sammatam braahmanendraanaam.

Both mean the same thing. He works in tandem with other scholars of his time. The society that values traditionally acquired knowledge, accepts his astronomical records and findings.

2.3.Lagadha attaches importance to that eternal system of knowledge called veda. He adores the vedic scholars.

vidvaan vedavit ashnute.

According to him, the fruits of the knowledge of astronomy are meant for vedic scholars. The entire subject is acknowledged as an ancillary subject of vedic studies. The main purpose of the knowledge of motion of celestial objects, according to Lagadha, is that it is essential for a vedic way of life.

2.4.He subscribes to the traditional view that each asterism is governed by a presiding deity.

agni: prajaapatissomo … naxatradevataa hyetaa:

It is seen that the entire list here is exactly as mentioned in Yajurveda, without even replacing them by their synonyms.

These observations suffice to conclude that Lagadha is a preserver of a hoary and flourishing tradition cherished and nourished by our ancient seers.

3.Lagadha, like other vedic seers, is devoted to God. This is evident from some of his passages shown below.

3.1.He starts his book with a benedictory verse of prayer.

pancasamvatsaramayam yugaadhyaxam prajaapatim
dinartvayanamaasaangam pranamya shirasaa shucih

Here the vedic deity prajaapati, in his capacity as the presiding deity of yuga, is saluted. The author bows his head to this form of God before embarking on his work.

3.2.He believes that one should be clean **(shuchi)**, before embarking on a task like this.
3.3.This devotion has percolated to his followers as well. The one who redacts the teachings of Lagadha, also salutes the diety of time while starting his work. This verse also forms a part of the book vedaangajyotisham. The verse is as follows:

pranamya shirasaa kaalam abhivaadya sarasvatiim.
kaalajnaanam pravaxyaami lagadhasya mahaatmanah.

We may note that in addition to the deity of time, the female deity of education, Sarasvati, is also worshipped here.

3.4.According to Lagadha, God functions through many gods. Here is a list of gods mentioned in his book: yugaadhyaksha prajaapati, kaala, sarasvati, chandra (with synonyms soma, indu) surya (with synonyms arka, ravi) vasu, tvashtaa, bhava, aja, mitra, sarpa, ashvinau, jala, dhaataa, ka:, agni, prajaapati, rudra (with synonym bhava), aditi, bruhaspati, sarpa, pitru, bhaga, aryamaa, savitaa, tvashtaa, vaayu, indraagnii, mitra, indra, nirruti, aapa:, vishvedevaa:, vishnu, vasus, varuna, aja ekapaat, ahirbudhnya, pusha, ashvini, yama.

4.Now it is time for us to provide samples from the main contents of the book. This will enable us to understand Lagadha as an astronomer. We mention four of the several topics that Lagadha dealt with . They are: 4.1. Units of time. 4.2.Time divisions of a yuga 4.3.A list of problems for which this book gives the methods of calculation. 4.4.a sample computation.

4.1.Here is a table proposed in this book:

kalaa dasha ca vimshaa syaat dvimuhuurtastu naadike. …
naadike dve muhuurtastu

5 gurvaksharas = 1 kaashthaa
124 kaashtaas = 1 kalaa.
(10+ 1/20) kalaas = 1 naadikaa.
2 naadikaas = 1 muhurta.
30 muhurtas = 1 day.
61 days = 1 ritu.
3 ritus = 1 ayana.
2 ayanas = one year.
5 years = 1 yuga.

Two remarks are now in order. These terms are completely vedic. Reference may be made to the upanishadic passage:

kalaa muhuurtaah aashtaashca ahoraatraashca sarvashah.
ardhamaasaa maasaa rutavassamvatsarashca kalpantaam.

Lagadha, who is fond of whole integers, and usually avoids fractions, has here made an exception. One naadikaa is made up of ten and one twentieth of kaashtaas. However he has deliberately avoided fractions in those parts of the table which more laymen would have to apply in their daily life. This is the reason why his year has 366 days, more than the actual (and known to him through vedas) 365 and a quarter.

4.2. How many periodical astronomical events take place in a yuga? According to Lagadha, there are

5 solar years
67 lunar siderial cycles
1830 days
62 synodic months
1860 tithis
135 solar nakshatras
1809 lunar nakshatras and
1768 risings of the moon.

Later interpreters have discussed about the accuracy of these assertions. These are rounded off integers of the actual numbers. For example, it is known that 62 synodic months are exactly 1830.8965 days; Lagadha has ignored the fractional part.

4.3. Here is a partial list of astronomical ideas described in Lagadha's book: The daily nakshatras and tithis with their ending moments, the hour-angle of the sun at the ends of parvas and tithis, the hour-angle of shravishtaa with the lagnas, etc, "have been given by ingenious rules" that enable us to calculate them easily everyday. A practical way of

measuring time is described. (Nowadays we have advanced clocks to do this job.) For each asterism, the presiding deity's name is mentioned. A list of fierce ones, and a list of cruel ones among these is given. (These may not have any value to the present day astronomers.) The number of risings of shravishtaa in a yuga is calculated as 1835. Some other ideas touched are: variation in the daytime, beginning of the yuga-period, tithis in which ayanas can begin, method to calculate the tithi in which vishuvas occur, method to calculate the part of the day in which parva ends, method to calculate the total number of parvas(full moon days) lapsed so far in a yuga, method to find the nakshatra at any parva, method to calculate the nakshatra at a given tithi, method to calculate the time of beginning of the nakshatra current at the end of a given tithi, method to calculate the part of the day at which a given tithi ends, method to calculate sun's nakshatra at any time, correction for the siderial day, need for two extra lunar months in each yuga, method to calculate tithis yet to elapse in a ritu, and so on.

4.4. Here is a sample of computation taught in this book: Multiply the tithis gone after a parva by 11. Add it to the parts of the nakshatra current at the end of the parva. Divide by 27. Take the remainder. Use it in the Jaavaadi series. This gives the nakshatra current at the tithi. (Translation following TSK)

5.Lagadha attaches great importance to the vedic rituals called yajnas. The three pillars of vedic spirituality are karma (rituals), jnaanam (knowledge) and bhakti (devotion). Lagadha takes care to highlight all these three in his book. We cite the four lines where yajnas are mentioned.. The first talks about the purpose of writing this book. The second is about one of the uses of deities of stars in the yajnas. The third is about the importance of these rituals in the vedic lore. The last one is about the importance of astronomy in these.

5.1.This book is written in order to help to determine the actual time of performance of vedic rituals.
Yajnakaalaarthasiddhaye.

5.2.The names of these deities serve one more purpose. The performer of yajna bears this name on that occasion.
Naxatradevataa etaa etaabhir yajnakarmani

5.3.The vedas have indeed been revealed for the sake of the performance of sacrifices. (All other uses of vedas are secondary.) **"Vedaa hi yajnaartham abhipravruttaa:"** The word hi here means that it is a well known fact.

5.4.It is he who knows astronomy that knows the vedic rituals. **yo jyotisham veda sa veda yajnaan.**

6.Lagadha is not merely a scientist, but also a poet. His work consists of 36 verses in one recension and 44 in another, many of the verses being common to both. As a poet he employs similes, metaphors, wordpuns, and the like to add charm to his scientific exposition that may otherwise become unappealing. Now we see some instances.

6.1.While delienating the importance of the subject of Astronomy among various ancilliary subjects of study in the vedic lore, he writes:

> **yathaa shikhaa mayuuraanaam naagaanaam manayo yathaa.**
> **Tadvad vedaangashaastraanaam jyotisham muurdhani sthitam.**

This verse has become very popular nowadays, because many authors have quoted it in their general books on ancient mathematics. Here two similes have been mentioned. First is that the crest is on the head of the peacocks.

The second is that a crown-jewel is on the head of the cobra. Similarly, Astronomy is "on the head" of the subjects of study. By these similes, he wants to convey three things: 1.Astronomy is the splendrous part among the entire study materials; after all, it is the study of shining objects in the celestial system, as are the crest of the peacock and the jewel on the serpant-hood. 2.It is the interesting and attractive part of the study. Just as the peacock is admired for its crest, and the serpant is admired for its jewel, education is admired because of this subject. 3.Just as the crest lies above all other limbs, and the hood-jewel lies above all other parts of the body, this subject is kept above all other subjects. In short, the three items of equality between the compared objects are splendour, beauty and placement.

6.2.The poet in Lagadha comes out more forcefully in the beginning, in the end and right in the middle. In the very first verse, the poet personifies the yuga. If the God is an embodiment of the yuga-period, then the sub-periods like years, seasons and days become His limbs.

6.3.A poet shines with a larger vocabulary. Moreover, the constraints of the verse-meter will force a poet to go for synonyms. Lagadha excels in this art. He uses the words nakshatra, ruksha, bha, and stru, synonymously. (By the by, the English word star may have come from the Sanskrit word stru with the same meaning.) He uses the words indu, paulastya, soma and chandramaa as synonyms for the moon.

6.4.As a good poet, he employs a variety of meters. Most of Lagadha's book consists of verses composed in anushtup-shloka meter, with eight syllables in each quadruplet. There is one verse in vidyunmaalaa meter, where all the thirtytwo syllables are long ones. The last verse is in indravajra meter. There is another with a slight variation thereof, called upajaati meter. His effort to a strict adherence to the metrical restrictions is clear from the phrase dinartvayanamaasaangam in the very first stanza. The meaning is: Day, season, ayana and month are the limbs. In prose, we prefer to rearrange them as day, month, season and ayana, in the increasing order of duration. Lagadha has changed this order in two occasions, just to suit the convenience of the meter. However, we come across some violations of meter-related rules, most probably because it has been mutilated by many writers over many centuries, with too many variations.

7.Lagadha has shown a good mastery over words, particularly the ones with double meanings. We explain this through four examples.

word	first meaning	second meaning
veda	revealed scriptures	knows
carita	motion	moved about
kaala	time	the god of time
parva	the full moon day	a fortnight

Occasionally these double meanings result in humour. However Lagadha's humour is always mild.

7.1.This is the last line of the work. **yo jyotisham veda sa veda yajnaan.** He knows the yajna-rituals, who knows astronomy. The word veda occurs here as a verb meaning "knows". In the same verse the first line is: **vedaa hi yajnaarthamabhipravruttaa:** Here the same word veda occurs as a noun. The sentences can be combined: One should know Astronomy, to understand yaaga, which is the very purpose of the vedas. The occurrence of the noun veda and the verb veda in a single verse contributes to a mild humour.

7.2.Lagadha uses the word carita in two meanings, that are closely related to each other. His verse is given below. The repetition if the first quadruplet again as the third, is meant to draw our attention to this pun.

Somasuuryastrucaritam vidvaan vedavidashnute.

Somasuuryastrucaritam lokam loke ca santatim

One who understands the motion of the moon, the sun, and the stars, attains the worlds of the moon, the sun and the stars. In this sentence, the object of knowledge coincides with the adjective of the world attained as its fruit. The charm in this coincidence is enhanced by the word caritam that qualifies both, in its two different meanings. If their motion is studied, the world in which they move about is attained. We may further add that in the same verse the word "world" is also employed twice, first to denote the other worlds like Suryaloka and Chandraloka, and next to denote the mankind in this world, acceptability by whom is mentioned here as a fruit of knowledge.

7.3.The word kaala usually denotes the Lord of death yama, probably because he keeps the record of time. Therefore when Lagadha salutes the Kaaladevataa in the beginning of his book, some commentators write that Kaaladevataa is none other than the Yamadharmaraaja. The existing koshas and dictionaries are in favour of this. Therefore a mild humour is felt in the following verse of vedaangajyotisham:

pranamya shirasaa kaalam abhivaadya sarasvatiim.

kaalajnaanam pravakshyaami

The meaning is: "After saluting kaala, I am going to describe the knowledge about time".

7.4.The word parva has its primary meaning as full moon day. It is employed in this meaning by Lagadha at least four times. But it also means the fortnight (consisting of fifteen days) as seen in the usage of the word parvasandhi. Lagadha accepts this meaning of the word parvan in one of his verses. But no humour results, because the two meanings are at two different passages. Similarly Lagadha uses the word Raashi in two meanings, heap and number, in two different contexts.

In summary, Lagadha takes advantage of word-puns also.

8.Lagadha writes cryptically at times. It helps him in achieving brevity, in enhancing curiosity, and in avoiding dullness. Occasionally however it baffles the commentators.

8.1.Look at this verse: **jau draa gha: khe …**These are the abbreviations that Lagadha uses for the names of the stars. All these are of single syllables. They are selected syllables of the full names, one for each star.

8.2.He uses very short words, with one or two syllables, profusely. Here is a list of some single syllable words employed in this book: dvi, tri, shat, sva:, syaat, syu:, tu, ca, hi, ka:, sa:, ya:, te, tat, yat, dyu, stru, bham, tau, dve, yaa:, saa.

8.3.There are many words or passages in Vedaangajyotisha whose correct meanings are still debatable. Here we cite a few. (1) In the line "Rridu pancadashaashtame" what is the meaning of the word ridu? Is there a mistake in this word? (2)In the line "suryaan maasaan shadabhyastaan" TSK has modified it as "staryaan maasaan". Which is the correct reading? Why? (3)In the stanza that starts with "dyu heyam.h" the researchers widely differ in interpreting. Whose interpretation is close to the text? (4) In the stanza starting with "syu: paadordham tripaadyaa yaa" which one is the correct reading? (5) For the words aavaapa and udvaapa, whose meaning is correct? (6)Has Lagadha used

bhutasankhyaa, as claimed by some? (7)Among the meanings provided to the word yugalabdham, which one is likely to be intended by the author? (8) In the line “yogam dinaikaadashakena tadvat” is anything else lurking, as suspected by TSK? And so on.

8.4.“The rules are couched in archaic, technical and terse language”, writes TSK. In the verse starting with “caturdashimupavasatha:” there is a clearly visible flaw of prosody. Is anything missing here?

9.For Lagadha, Arithmetics and Astronomy are intimately related.

9.1.The word ganitam for arithmetics and the word jyotisham for astronomy are mutually substitutable in one context. In the passage **tadvat vedaanga shaastraanaam jyotisham murdhani sthitam,** Lagadha himself has allowed a modification **ganitam murdhani sthitam.** This shows that Lagadha considers both Mathematics and Astronomy as the most important among the ancillary subjects of vedic studies (the others being, phonetics, grammar, prosody, etimology and ritual science.) Does he mean that these two are one and the same? Not likely. Because in his tradition, they have been listed separately. As early a treatise as Taittiriya Braahmanam, mentions that an astronomer and a mathematician are different as professionals: **prajnaanaaya naxatradarsham** and **viinaavaadam ganakam giitaaya** are the lines there. If they are different, are both counted among the vedaanga subjects? It cannot be so, because the number of vedaanga subjects is fixed as six. There are two ways to resolve this problem. First view: For Lagadha, ganitam and jyotisham are synonyms. In those days, the entire mathematical canopy was meant for astronomy. Second view: Mathematics is not the same as Astronomy. Even in Chaandogya upanishat, raashividyaa and nakshatravidyaa are mentioned separately. Therefore these two verses of Lagadha, convey in two different ways that both Mathematics and Astronomy are important subjects. How can the same set of words be subjected to two interpretations? There again lies the cleverness of the poet. Let us explain. Astronomy is one of the six vedaangas. Mathematics is not so. But it is developed in many of the vedaangas. For instance, in prosody, Theory of Binary Number System is developed. In Kalpa (ritual Science) the theorem of Hypotenuse (nowadays attributed to Pythagorus) is explained. In jyotisham rudiments of Trigonometry are applied. This list can be extended. All these happened in India many centuries before Christ. Bodhayana’s algorithm for rational approximation of irrational numbers, Pingala’s dealings with the number zero, all these belonged to that ancient era, in different vedaangas. Therefore it is correct if Lagadha claims that in every ancillary subject of the vedic literature, Mathematics is kept at the top position. This is similar to a statement of a Nobel Laureate that in every branch of science, only that part is proper science, which has been illustrated mathematically. The verse “yathaa …ganitam murdhani sthitam” is therefore interpreted as follows: Just as many peacocks are adorned by the crests on their heads, and just as many king-cobras are adorned by jewels on their hoods, many vedaangas are adorned by a top position for Mathematics. Contrast this with the previous interpretation reproduced below: Just as a peacock carries a crest on its head, and just as a serpant carries a jewel on its hood, so is Astronomy lying on the top of vedaangas. The main contrast is as follows: (1)If a peacock is likened to the full body of vedaanga subjects, then its crest is likened to Astronomy. (2)If there are many peacocks that are likened to many vedaangas, (one each), most of them are adorned by crests that are likened to Mathematics. The difference between these two statements is crystal-clear. Because Lagadha wants to make both these assertions, he has two versions of this verse, one in Rik-jyotisham, and the other in Yajur-jyotisham (the only difference being the replacement of ganitam by jyotisham and viceversa).

9.2. Occasionally general arithmetical problems are discussed. For instance, the rule of three, is explained by a verse:

ityupaayasamuddesho bhuuyopyenam prakalpayet.
jneyaraashigataabhyastam vibhajejjnaanaraashinaa.

Its meaning is: The known result is to be multiplied by the quantity for which the result is wanted, and divided by the quantity for which the known result is given.

9.3. In Lagadha's book on Astronomy, Arithmetics plays a major role. The following table of technical terms gives a rough idea of items used:

raashi	Number (positive integer)
Yoga	Addition
vihina	Subtracted by
Una,shesha	Remainder
Gunita, abhyasta	Multiplied by
Vibhajanam	Division
Bhinnam	Fraction
Bhinna-apanaya	Rounding off to an integer.
Naadikaa-pramaanam	Volume size to define a time-unit called naadikaa
Upaaya-samuddesha	Rule of three
ekaantara	Alternating
heyam	To be omitted
jaavaadi	Using addition modulo five.
labdham	Quotient

9.4.Incidentally the units of volume are also discussed. Palam is a unit of weight. 50 palams of water make one Adhakam. 4 Adhakams make one dronam, whereas one fourth of an Adhakam is callaed a kudavam. 13 kudavams make a naadika. The same word naadikaa is used both as a volume unit and as a time unit. This is because, the naadikaa time is the time required by a naadikaa-volume of water in a specially designed vessel (whose description is to be taken from elsewhere) to drain out completely. This common unit name is still in use in both the senses.

10.Last but not the least, is the fact that Lagadha is a leader, with many followers, interpreters, commentators and admirers. We now make four remarks on this aspect.

10.1.In this book itself, we find a verse, probably composed by one of his students. **kaalajnaanam pravakshyaami lagadhasya mahaatmanah**. Here the word mahaatmaa means an eminent person. The author of this shloka is acknowledging that the subject matter of this book is the discovery of the eminent sage Lagadha.

10.2.Here are some quotes about this book of Lagadha.

"These verses would have remained obscure and unexplained forever if we have not received light from an unexpected quarter, …,Surya prajnapti and Jyotishkaranda".-R.Shamasastry in the introduction of his book.

"In the course of oral transmission of the text through several generations of the adhvaryu priests over a period of nearly 3500 years, it is quite natural that many verses came to be handed over to the present generation often in erroneous and corrupted forms" – S.Balachandra Rao in his book "Indian Mathematics and Astronomy".

"Vedaangajyotisha, the astronomical auxiliary of the vedas, is the earliest Indian text devoted exclusively to the treatment of astronomy" – K.V.Sarma in his preface to the edition by T.S.Kuppanna Sastri.

10.3..This book has attracted the attention of many scholars in the later period. Here is a partial list, based on the information provided by the last one mentioned below.

Period	Book/Author	Remarks
--	Artha sastra	Follows it for almanac making.
--	Paitaamaha siddhaanta	Refers to Lagadha's system.
B.C.	Suryaprajnapti, a Jain work	Almost reproduces in Praakrit language.
1834	Weber	First edition of both recensions together.
1877	Thibaut	Deciphers a few difficult verses
1896	S.D.Dikshit	Marathi interpretation of some verses.
1907	Lala Chote Lal	Own interpretation to all verses.
1907	Sudhakara Dvivedi	Edited with an old Sanskrit commentary of Somaakara.
1914	Bal Gangadhar Tilak	Criticisms and suggestions for interpretation.
1916	Samikkannu Pillai	Discusses the Calendar part only.
1936	Shamasastry	Sanskrit Commentary and English translation.
1984	Kuppanna Sastri	Critical edition with almost thorough translation and notes.

10.4.As a good populariser of his subject of study, Lagadha mentions many advantages of studying his book. (1)Unless performed in the prescribed correct timings, the yajnas will not be fruitful. Astronomical knowledge provided in this book is essential for this purpose. (2)**"jyotishaam ayanam punyam"**. It is meritorious. One obtains a lot of punyam by studying this. (3)This subject is on the top of the six vedaangaas. (4)It is so respectable a subject that one has to be pure and clean while studying it. **"pranamya shirasaa shuci:".**(5)One who learns the celestial motions will be doubly rewarded. In this world he will be bestowed with a continuing progeny. Later he will attain the worlds of Chandra and Surya.

Conclusion. Several ancient Indian Mathematicians were simultaneously poets, cryptic writers, researchers, expositors, teachers, traditionalists and leaders. Lagadha is one among them.

Mathematics in The Vedas

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PRELUDE

1.0. Even though the age of the Vedas is a subject of continuous research even today, it is undisputed that the Vedas are the earliest records of human wisdom which are being handed down to humanity through oral tradition of knowledge transmission. What ever may be the thinking of the historians in 19^{th} and early 20^{th} century, new researches which peeped into the internal evidence of the Vedas firmly establish the age of Vedas to be 6000 BC – 19000 BC, basing on the astronomical references available in the literature.[1]

1.1. Surprisingly, such a prehistoric bulk of records carries intricate scientific statements pertaining to several branches of science such as mathematics, medicine, astronomy, cosmology, botany etc. The purpose of the present article is to bring out the mathematical acumen, branch wise that can be evidently inferred from the outer meaning of the Vedic sentences, without touching those complicated implications that can be derived through philosophical inquiry.

2.0 ARITHMATIC AND VEDAS

We know that mathematical knowledge initiates its progress through the mechanism of counting and numbering. In the annals of history of mathematics, the process of counting and numbering had its first leap forward with the famous book "Calculus of Sand" written by Archimedes of 3^{rd} century BC. This could not progress well and expand its wings into bigger multiplications until the concept of zero entered into Europe, through the Arabic channels. It is now an accepted fact that the marvelous concept of zero was invented by Indians and from there it migrated to the Arabs. We shall take up this concept a bit latter in this article and we shall now focus on the other digits of the number system, and then pass on to fractions, series, zero and infinity, available in the Vedas.

2.1. Digits and number system

The concept of digits, to the extent of several millions, can be clearly seen in all the Vedas.[2] We shall take one quote from each Veda, except SamaVeda, since Sama Veda is almost a musical modulation of Rig Veda.

2.1.1. Rig Veda

आ विंशत्या (२०) त्रिंशत्या (३०) याह्यर्वांडा चत्वारिंशता (४०) हरिभिर्युजानः
आ पंचाशता (५०) सुश्थेभ्रिरिन्द्रा षष्ट्या(६०) सप्तत्या (७०) सोमपेयम् ।।
आशीत्या (८०) नवत्या (९०) यांह्यर्वांडा शतेन (१००) हरिभिरुह्मयमानः
इथं हिते शुन्होत्रेधु सोम इन्द्र त्वासा परिषिक्तो मदाया ।। (ऋग्वेद / मण्डल -२ / सूक्त १८ / मंद्र ५-६)

ā vinśatyā (20) trinśatyā (30) yāhyarvāṇḍā catvārimṣatā (40) haribhiryujānaḥ
ā pañcāśatā (50) suśthēbirindrā ṣaṣṭyā(60) saptatyā (70) sōmapēyam ..
āśītyā (80) navatyā (90) yāṃhyarvāṇḍā śatēna (100) haribhiruhmayamānaḥ
ittaṃ hitē śunhōtrēdhu sōma indra tvāsā pariṣiktō madāyā .. (ṛgvēda / maṇḍala -2 / sūkta 18 / mandra 5-6)

This hymn clearly mentions

Vimśatya	*-which means*	*20*
Trimśata	*-which means*	*30*
Caturvimśata	*-which means*	*40*
Pancaśata	*-which means*	*50*
ṣaṣṭya	*-which means*	*60*
Saptatya	*-which means*	*70*
Asitya	*-which means*	*80*
Navatya	*-which means*	*90*
śatena	*-which means*	*100*

This is not an occasion for the hymn to mention the number system as it is. The occasion demands the series to start from 20. The point which deserves our attention is the word ‘trimsata’ which is the combination of two words ’tri’ and ’imsat’, which means 3 multiplied by 10.

Similarly ‘chatvari’ ‘imsat’ implies multiplication of ‘chatvari (4)’ and ‘imsat (10)’

‘pancaśat’	implies	5x10
‘śaśṭi’	implies	6x10
‘saptati’	implies	7x10
‘asiti’	implies	8x10
‘navati’	implies	9x10

‘śata’ (100) is a new name.

The above observation implies that the Vedic people had the knowledge of multiplication as well as the number system, with 10 as its base.

2.1.2 Yajur Veda

The existence of bigger numbers in series can be seen in Yajur Veda.

एक (१) च दश (१०) च शतम (१०^{२}) च, शतम् (१०^{२}) च सहस्त्रम(१०^{३}) च् सहस्त्रम् (१०^{३}) च अयुतम् (१०^{४}) च, अयुतम्(१०^{४}) च नियुतम्(१०^{५}) च नियुतम् (१०^{५}) च, प्रयुतम् (१०^{६}) च , अर्ब्रुदं (१०^{८}) च, न्यर्ब्रुदं च, समुद्रश्च, मध्यम्(१०^{१६}) च, अन्तश्च, परार्द्धश्च्(१०^{१७}) एताः मे अग्ने इष्टिकाः देनवः सन्तु, अमुत्रामुष्मिन् लोके । (यजुर्वेद / अध्याय १७ / कण्डिका २)

ēka (1) ca daśa (10) ca śatama (10^2) ca, śatam (10^2) ca sahastrama(10^3) c sahastram (10^3) ca ayutam (10^4) ca, ayutam(10^4) ca niyutam(10^5) ca niyutam (10^5) ca, prayutam (10^6) ca , arbudaṃ (10^8) ca, nyarbudaṃ ca, samudraśca, madhyam(10^{16}) ca, antaśca, parārddhaśc(10^{17}) ētāḥ mē agnē iṣṭikāḥ dēnavaḥ santu, amutrāmuṣmin lōkē . (yajurvēda / adhyāya 17 / kaṇḍikā 2)

This hymn indicates a series of numbers which starts with 1, the next numbers being 10, 100, 1000, … up to 10^{17}. This series runs in multiples of 10 and is in the form of a geometric progression of the type – a, a^2, a^3… etc. The concept of power to 10 does not appear, but the series is given perfectly with individual names to each of the multiples. This again establishes that the Vedic people had a number system, with 10 as its base.

2.1.3. Atharvan Veda

The following hymn from Atharvan Veda supports the above concept.

एका(१) च मे दश(१०) च मेपवक्तार औषधे ।। द्वे(२) च मे विंशतिश्च (२०) मेपवक्तार औषधे।। तिश्तरश्य (३) मे त्रिंशच्च (३०) मेपवक्तार औषधे चतस्त्रश्च (४) मे चत्वारिंशच्च(४०) मेपवक्तार औषधे।। पंच(५) च मे पंचाशच्च(५०) मेपवक्तार औषधे षट् (६)च मे षष्टिश्च (६०) मेपवक्तरा औषधे ।। सप्त(७) च मे सप्ततिश्च(७०) मेपवक्तार औषधे अष्ट (८) च मे शीतिश्च (८०) मेपवक्तार औषधे नव(९) च मे नवतिश्च(९०) मेपवक्तरा औषधे दश(१०) च मे शतं(१००) च मेपवक्तार औषधे शतं(१००) च मे साहस्त्रं (१०००) च मेपवक्तार औषधे ऋतजात ऋतावरि मधु मे मधुला करः (अथर्ववेद /काण्ड ५ / सूक्त १५ / मंत्र १-११)

ēkā(1) ca mē daśa(10) ca mēpavaktāra auṣadhē .. dvē(2) ca mē vinśatiśca (20) mēpavaktāra auṣadhē.. tiśtaraśya (3) mē triṃśacca (30) mēpavaktāra auṣadhē catastraśca (4) mē catvāriṃśacca(40) mēpavaktāra auṣadhē..pañca(5) ca mē pañcāśacca(50) mēpavaktāra auṣadhē ṣaṭ (6)ca mē ṣaṣṭiśca (60) mēpavaktarā auṣadhē .. sapta(7) ca mē saptatiśca(70) mēpavaktāra auṣadhē aṣṭa (8) ca mē śītiśca (80) mēpavaktāra auṣadhē nava(9) ca mē navatiśca(90) mēpavaktarā auṣadhē daśa(10) ca mē śataṃ(100) ca mēpavaktāra auṣadhē śataṃ(100) ca mē sāhastraṃ (1000) ca mēpavaktāra auṣadhē ūtajāta ūtāvari madhu mē madhulā karaḥ (atharvavēda /kāṇḍa 5 / sūkta 15 / mantra 1-11)

The specialty of this hymn is that it gives digit - linkage between 1and 10, 2 and 20, 3and 30, 4 and 40, 5 and 50, 6 and 60, 7 and 70, 8 and 80, 9 and 90, 10and 100 and it jumps to 100 and 1000.

From this it can be re confirmed that 10 was used as base for the number system. More over, the internal consistency with respect to number system of the Vedic hymns can be well established by this.

2.2. Series

एका च मे तिस्रश्च मे पंच च मे सप्त च मे नव च मे एकादश च मे त्रयोदश च मे पंचदश च मे सप्तदश च मे नव दश च म एकविग्ं शतिश्च मे त्रयोविग्ंशतिश्च मे पंचविग्ं शतिश्च मे सप्तविग्ं शतिश्च मे नवविग्ंशतिश्च मे एकत्रिग्ं शच्च मे त्रयस्त्रिग्ं शच्च मे चत्रसश्च मेष्टौचमे द्वादशचमे षोडशचमे विग्ंशतिश्चमे चतुर्विग्ं शतिश्च मे ष्टाविग्ंशतिश्चमे द्वात्रिग्ंशच्चमे षट्त्रिग्ंशच्चमे, चत्वारिग्ं शच्चमे चतुश्चत्वारिग्ंशच्च मे ष्टाचत्वारिग्ं शच्चमे
(यजुर् - चमकध्याय - ११ स्टानजा)

ēkā ca mē tisraśca mē pañca ca mē sapta ca mē nava ca mē ēkādaśa ca mē trayōdaśa ca mē pañcadaśa ca mē saptadaśa ca mē nava daśa ca ma ēkavigṃ śatiśca mē trayōvigṃśatiśca mē pañcavigṃ śatiśca mē saptavigṃ śatiśca mē navavigṃśatiśca mē ēkatrigṃ śacca mē trayastrigṃ śacca mē catrasaśca mēṣṭaucamē dvādaśacamē ṣōḍaśacamē vigṃśatiścamē caturvigṃ śatiśca mē ṣṭāvigṃśatiścamē dvātrigṃśaccamē ṣaṭtrigṃśaccamē, catvārigṃ śaccamē catuścatvārigṃśacca mē ṣṭācatvārigṃ śaccamē
(yajur - camakadhyāya - 11 sṭānajā)

This hymn of Yajur Veda gives a series of numbers

(a) 1, 3,5, 7,9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33

and (b) 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48 which is of the form a, a+d, a+2d …. etc.

We can see that these two series are in arithmetic progression. We can not proclaim their knowledge about the formula to find the n^{th} term of the series and the sum of n terms. The same is true in the case of the numbers in geometric progression, which was quoted in para 2.1.1

2.3 Fractions

The above facts, quoted so far, will establish the presence of natural numbers, that too very big numbers, in the Vedic literature. Now we shall have a look at the fractions. In the available literature (since a lot of Vedic literature was lost), not many direct references pertaining to fractional numbers are available. How ever, the concept of division and fractions can be clearly understood in the following Rig Vedic hymn.

ज्येष्ठ आह चमसा द्वा (१/२) करोति कनीयान् त्रीन् (१/३) कृणवामेत्याह
कनिष्ठ आह चतुरस्करोति (१/४) त्वष्ट ऋभवस्तत् पनयद् वचो वः (ऋग्वेद/मण्डल ४/सूक्त ३३ / मंत्र ५)
jyēṣṭha āha camasā dvā (1/2) karōti kanīyān trīn (1/3) kūṇavāmētyāha
kaniṣṭha āha caturaskarōti (1/4) tvaṣṭa ūbhavastat panayad vacō vaḥ (ṛgvēda/maṇḍala 4/ sūkta 33/ mantra 5)

The hymn speaks of dividing a glass of drink in to two, three, four parts, However, whether they had a system of representing the fractions in the form of a numerator and a denominator is uncertain. Still, we can not totally deny that, since the concept of infinitely big and infinitesimal entities were mentioned in the Vedas, at different contexts. We shall discuss the fractions, once again, in the context of Astronomy.

2.4. Special digit zero

The above mentioned mathematical exercise can not be imagined if the concept of zero and the place value system of numbers were not available with them. Even though direct statements in this regard are not available, we can not deny their presence because of the consequent progress in other branches of mathematics, such as geometry and astronomy which we shall study a bit latter. Zero as a member of the numerical system and zero as a symbol of emptiness, both can be inferred in the philosophical context of the Vedic literature, but that is beyond the scope of this article.

2.5. Infinity

As mentioned already, the concept of infinitely big (Ananta) and infinitesimal (Paramanu) entities are very much available in the Vedas and this can be clearly understood in the context of philosophy. Taittiriya Upanishad says aṇōraṇīyām mahato mahiyan' which means that pervading Almighty is as big as 'Ananta' (infinity) and as small as Paramanu (atom). The same concepts were adopted in mathematics as infinity and zero. Out of the two, the mathematical implications of infinity can clearly be seen in the following hymn of Yajur Veda.

ॐ पूर्णमदः पूर्णमिद पूर्णात् पूर्णमुदच्यते पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते (यजुर्वेद / अध्याय ४/ कण्डिका ३)
ūṁ pūrṇamadaḥ pūrṇamida pūrṇāt pūrṇamudacyatē pūrṇasya pūrṇamādāya pūrṇamēvāvaśiṣyatē
(yajurvēda / adhyāya 4/ kaṇḍikā 3)

This hymn means: Infinity comes out of infinity and when infinity is subtracted from infinity, the result remains as infinity. This, clearly, is similar to the modern concept of infinity.

Similar concepts on zero are seen in the Tantric and Puranic literature of India, which belong to the succeeding periods of Vedas. Since the scope of this article does not cover those literatures, let us stop at this juncture and proceed to have glimpses at the other mathematical branches of Vedic literature.

3.0 GEOMETRY IN VEDAS

While arithmetic deals with mathematics of numbers, geometry deals with mathematics of space, which precipitates itself in the form of lines and curves. The simplest form of polygon can be taken as triangle, since 3 is the minimum number of sides with which a polygon can be drawn. The name of the triangle (Tribhuja) can be seen in the following lines from Atharvan Veda, in a non mathematical context.

यो अक्रन्दयत् सलिलं महित्वा योनिं कृत्वा त्रिभुजं शयानः ।
वत्सः कामदुघो विराजः स गुहा चक्रे तन्वः पराचैः ।। (अथर्व - VIII- ९-२)
yō akrandayat salilaṃ mahitvā yōniṃ kūtvā tribhujaṃ śayānaḥ .
vatsaḥ kāmadughō virājaḥ sa guhā cakrē tanvaḥ parācaiḥ .. (atharva - VIII - 9-2)

The Almighty made a triangle (Tribhuja) with earth, the intermediate space and the heaven as its sides and the generated multiple forms of beings, in a mystic way.

4.0. ASTRONOMY AND VEDAS

When the two dimensional geometry is extended to three dimensions, a new subject, called Astronomy, manifests itself. This branch of mathematics is seen extensively in all the Vedas and more so in Rig Veda. I-SERVE published a thorough work on this subject under the title 'Indian Astronomy in Pre Siddhantic Period' written by Dr. K. D. Abhyankar, a retired professor from Osmania University, Hyderabad. Just as a glimpse, we shall see an interesting hymn from Atharvana Veda.

द्वादश प्रधयश्चक्रमेकं त्रीणि नाभ्यानि क उ तच्चिकेत । (अधर्व - X- ८-४)
dvādaśa pradhayaścakramēkaṃ trīṇi nābhyāni ka u taccikēta . (adharva - X - 8-4)

The context of this hymn is the description of the formation of a year, a month and a day. The hymn says that the orbit of earth is in the form of a circle which has three centers. In Sanskrit, 'circle with three centers' is the technical name for an elliptical orbit. Even in modern mathematics, elliptical orbit has three centers, one main center and two epi centers. This example is enough to show that the Vedic Astronomers had enough insight into 3-dimensional geometry and its related branches.

In another hymn, the concept of Adhikamasa (13th month[3]), which is needed to establish correlation between the solar year and lunar years, is proposed.

4.1. Astronomy also derives smaller fractions of time units, which are involved in the calculations of star timings etc., and establish smaller units of time (kalä kästhä), etc which can be compared to seconds and subdivisions of seconds. Such intricate astronomical conclusions are not possible in the absence of the knowledge of fractions and their mathematical operations.

The calculations of eclipses and their repetitions in fixed periods (which are extensively available in Rigveda[4]), involve multiplications and divisions with numericals containing 12 to 15 digits. Such mathematical operations are not possible in the absence of a number system based on place value.

5.0. In view of the above discussions, which are confined to limited quotations containing only outlines of the subject, we can estimate the depth of mathematical expertise of the Vedic period.

END NOTES

1. (a) The Arctic Home in the Vedas by Sri Balagangadhara Tilak
   (b) The Orion and Arctic Home by Dr. Jacobi & Tilak (These authors took Vedas to 6000 B.C)
   (c) Celestial key to Vedas: Discovering the origins of the World largest civilization, Publisher – Inner Traditions, by B.G. Siddharth, (He took Vedas to around 10,000 B.C)

2. The Veda was divided into four parts by sage VedaVyasa in around 3000 BC. The parts are called 1.Rig Veda 2.Yajur Veda 3.Sama Veda and 4. Atharvan Veda. Some of the European scholars opined that the Vedas have come up in a sequential manner in the order mentioned above. Some others denied it on the ground of several internal evidences, which include the fact that the name of Atharvan , Sama and YajurVedas are mentioned in Rig Veda itself. How ever, for our present study, this historical discussion is not so important, because we treat the whole Vedic literature as a single unit and proceed with our study of the mathematical concepts available therein.

3. अहोरात्रैर्विमितं त्रिंशदंशगं
   त्रयोदशं मासं यो निर्मिमीते (अधर्वा XIII, ३-८)
   ahōrātrairvimitaṃ triṃśadaṃśagaṃ
   trayōdaśaṃ māsaṃ yō nirmimītē (adharvā XIII, 3-8)
   (The sun generates a 13th month, which has 30 sets of days and nights on its wings)

4. A lot of work has been done on this aspect of Vedic Astronomy by Dr. Subhash Kak of Oklahoma State University , USA and Dr. R.N.Iyangar, Raja Ramanna Awardee of Indian Institute of Science, Bangalore , India.

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Sulvasutra in a Nutshell

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Even though Indians regard four Vedas and related literature which includes various Samhitâs, Brâhmaṇâs, Âraṇyakâs and Upaniṣads as the expiration of the supreme soul, (*asya mahato bhûtasya niśvasitametad yad ṛgvēdō yarurvēdaḥ sāmavēdaḥ atharvāngirasaḥ*) the study of 6 vedâñgâs (limbs or members of Vedâpuruša) namely śikša: phonotics, kalpa: rites or rituals, vyākaraṇa: grammer, nirukta: etymology, jyotiša: astronomy and chanda: prosody is considered mandatory to understand the exact and flawless meaning of Vedâs. Amongst these vedâñgâs, Kalpa is considered to be the hands and jyotiša as the eyes of the vedapuruša.

Kalpa firstly means a sacred precept, law, rule, ordinance, manner of acting or proceeding practice especially related to Vedâs (*kalponâma vidhih*) [1] and secondly it means that which prescribes the rituals and the rules for ceremonial or sacrificial acts (*kalpante samarthyante yajjña yâgâdi prayogah yatra sa kalpah*). The Kalpa sûtra of each Vedic Samhitâ is subdivided into three parts namely śrauta sûtrâs, Dharma sûtrâs and Gṛhya sûtrâs. Śulva Sûtra is either considered as part of corresponding Śrauta sûtra or fourth part of corresponding Kalpa sûtra.

The meaning of the word śulva or śulba or Rajju is cord rope or string and it is derived from basic root Śulv or Śulb meaning "to mete out " or "to measure" and hence its etymological significance is "measuring" or "act of measuring". B. Datta has elaborated four meanings of the same word śulba as (1) mensuration – the act and process of measuring (2) line (or surface) – the result obtained by measuring (3) a measure – the instrument of measuring (4) geometry – the art of measuring. He further concluded that earliest Hindu name for geometry was Śulba.[2]

The meaning of the word sûtra which is applicable here is "a key", "a formula", "a short rule" or "aphorism". Sûtravid (expert in formulating the sutra) has assigned following six characteristics to sûtra. (1) swalpâkšaram: having minimum number of words, (2) asandigdham: unambiguous, (3) sâravad – essence or summarized, (4) vishvatomukham – applicable to all cases or examples, (5) astobham – free from unnecessary explanations and (6) anavadyam – faultless or irreproachable.

With these meanings and arrangements one naturally expects as much number of Śrauta sûtrâs and Śulva Sûtras equal to thousands of Vedic cults. However only 13 Śrauta Sûtrâs and 9 Śulva Sûtras are available. The samhitâ wise distribution of them is as follows.

No	**Samhitā**	**Śrauta Sūtrās**	**Śulva Sūtras**	**No**	**Samhitā**	**Śrauta Sūtrās**	**Śulva Sūtras**
1	Ṛg veda	Śakhāyana Āvalāyana	Not available	**3**	Atharva veda	Vaitāna	Not available
2	Yajur veda	Kātyāna Baudhāyana Āpastamba Mānava Hiraṇyakeśhi Bhāradvāja Vaikhāsana	Kātyāna Baudhāyana Āpastamba Mānava Hiraṇyakeśhi Vādhula Laugākśi Varāha	**4**	Sāma veda	Maśaka Latyāyana Dākśyayana	Maśaka

Remarks – (1) Even though two Śrutasûtras related to Ṛgveda and one to Atharvaveda samhita is available, not a single Śulvasûtra attached to them is found. (2) Three Śrutasûtras attached to Sâmaveda Samhitâ are found but only one Śulvasûtra is refered. (3) Kâtyâyana Śrutasûtra and Śulvasûtra both are attached to Śuklayajurveda Samhitâ (4) Baudhâyana, Âpastamba, and Hiraṇkeśi Śrutasûtra and Śulvasûtra are attached to Kṛṣṇ a Yajurveda Taittirîya Samhitâ. Vâdhula Úulvasûtra of which Śrutasûtra is not found is attached to same Samhitâ. (5) Mânava Śrutasûtra and Śulvasûtra is attached to Kṛṣṇ a Yajurveda Maitrâyaòi Samhitâ. Varâha Śulvasûtra of which Śrutasûtra is not found is also attached to same Samhitâ. (6) Laugakśi Śulvasûtra of which corresponding Śrutasûtra is not available is attached to Kṛṣṇ a Yajurveda Kaṭhaka Kapisthala Samhitâ. (7) The relation of Śulvasûtra to corresponding Śrutasûtra and Samhitâ clearly focus on the very purpose for which they were composed. And the purpose is to meet the technical requirements of various meticulous constructions for successful yajñya. (8) Keeping in mind the attachment and the sûtra character of Śulvasûtras one need not wonder why the texts related to geometry are composed in aphoristic style and should not complaint or critisise their codified or cryptic nature.

Time Span of śulvasûtras

Majority of the Indian and many western scholars assign a period later than 800 BC to Śulvasūtra literature. Some of them are mentioned here.

(1) Sen and Bag in their book [3] suggested a period of 800 to 500 BC with a remark that "We are still far away from narrowing the date range by centuries and putting the early sûtra works on a firmer chronological basis". (2) Chronology committee patronized by INSA (1950) recommended a period 500 BC and later for Śulvasûtras. (3) Svami Satya Prakash Sarasvati in his book [4] agrees with the period 800 BC to 200 BC for Śulvasūtras. (4)T. A. Sarasvati Amma in her book [5] remarked that "— c 800 BC is a more probable date for the codification of sûtras". (5) R. C. Gupta in his paper [6] says 'They are variously dated and their exact time of composition or compilation is controversial. The Baudhâyana Śulba Sûtra is the oldest of them and is generally placed between 800 B.C. and 500 B.C.' (6) Venugopal Heroor in his book [7] adopted ten stages of development of mathematics in India and put the age of vedângas or Sûtra period 800 BC to 200 BC. Thus historians and scholars seems to be agree on the time span of 800– 500 – 200 BC for the codification of Śulvasûtras with clear understanding that Śulvasûtras are much older than their date of composition. B Datta said that " We can indeed trace most of the matters contained in Śulba to earlier Brâhmaṇâs and Samhitâs"[8]. For the detailed discussion on this aspect refer [9].

Recent researches strongly suggested different time span for Śulvasûtras and early Vedic literature. Abraham Seidenberg's research work [10] led to following conclusions.

(1) The common source of Pythagorean and Vedic mathematics is to be sought either in the Vedic mathematics or an older mathematics very much like it. (2) The view that Vedic mathematics is a derivative of Old-Babylonia having been rejected; a common source for these mathematics, different from Old-Babylonia of 1700 BC was indicated. (3) Thus what are regarded as the two main sources of Western mathematics, namely Pythagorean mathematics and Old-Babylonian mathematics of 1700 BC, both flow from still older source. (4) Hence we do not hesitate to place the Vedic alter period rituals, or more exactly, rituals exactly like them, far back of 1700 BC. (5) The elements of ancient geometry found in Egypt and Babylonia stem from system of the kind observed in sulbasutras. Vander Warden Book 'Geometry and Algebra in Ancient Civilization' (1983) takes a similar view.

Following Seidenberg's work and his own archeological studies N. S. Rajaram in his paper [11] says (1) It was in fact my study of Seidenberg's work that led me to conclude that Harappan civilization must prsupose knowledge of the mathematics of the Baudhayana Sulba and therefore correspond to Sutra period. (2) Harappans were part of the Vedic culture. — The sulbas contain technical instruction for the design and construction of various altars; many

examples of such altars have indeed been found among the Harappan ruins from the borders of Iran to Lothal in Gujrath. — Thus there can be hardly any doubt at all that the mathematics of the Sulbas must already have been in existence by the time the great Harappan settlements began to be planned. This means that the earliest layer of the Sulba, such as the Sulba of Baudhayana must have come to being not much later than 3000 BC. This early date for the Sutra literature is supported also by astronomy.

Another work based on Seidenberg work and lots of historical data, V. Laksmikantam and S. Leela authored the book "The Origin of Mathematics" where they refuted convincingly the view held by most of the scholars that mathematics originated in Greece and not in India. They further showed that the source of geometric algebra is Shulva Sutras and also refutes the popular claim that Shulva Sutras provide only rule without any proof. They put Birth of Baudhayana at 3200 BC. [12]

This author is of the view that we must accept latest views based on historical truths archeology, astronomy, Indo French field study and French SPOT satellite for further references.

Content of Śulva Sūtras

Four Śulvasūtras namely Baudhâyana, Âpastamba,, Kâtyâna and Mânava are most significant with respect to content hence they were extensively studied and commented upon by Indian and western scholars. Baudhâyana Śulvasūtra (BSS) is the oldest one has 21 parts and 295 passages; Âpastamba Śulvasūtra (ASS) contains 21 parts and 202 passages; Kâtyâna Śulvasūtra contains 6 parts and 67 passages and Mânava Śulvasûtra has 16 parts and 228 passages. This division is according to the Sen and Bag's book [13] which is refered throughout this article.

The very purpose of Śulvasûtras is to provide technical assistance to design and construct different Vedis and Citis (fire altars) for successful yajñas (sacrifices) we find discussion largely on the same with required instruments and units of measurements. Numerous geometrical constructions, transformations and combination of areas are extensively discussed. Basic geometrical ideas, concepts, properties, propositions or theorems are either explicitly stated or one can guess implied use of many such principles. Further we observe use of fractions and surds, indeterminate equations etc in the scripts. Now we discuss all these subject matters briefly.

Instruments –

Following instruments are found used for design and construction purposes.

(1) Rajju: Rope or cord (2) Veṇu or Vamśa: Bamboo rod or Cane stick (3) śamya: Pin or yoke pin (4) Śanku: Pole or Peg (5) Sphya: Wooden rod cone shaped at one end (6) Rods representing units of lengths like vyâma, artini, puruṣa etc. (7) The description of instrument having three holes on a bamboo rod at specified distance and its use in drawing circles and squares can be compared in limited sense with modern geometrical compass. [14]

Units of Measurements -

Various units of linear measurements are found applied in the Śulvasūtras. Surface and volume units are expressed in terms of linear units only. The basic unit of linear measurement is angula. The distance between the bottom of little finger and fore finger is considered to be 4 angulas which is assumed equal to 3 inches approximately [15]. Total sixteen different units of linear measurement used in the BSS are noted in the following table along with conversion in anguls and inches.

S. No.	Unit	angulas	inches	S. No.	Unit	angulas	inches
1	angula	4	3	**9**	bāhu	36	27
2	kšudrapada	10	15/2	**10**	śamya	36	27
3	prādeśa	12	9	**11**	yuga	86	129/2
4	Vitasti	13	39/4	**12**	vyāyama	96	72
5	pada	15	45/4	**13**	akša	104	78
6	aratni	24	18	**14**	Vyāma	120	90
7	prakrama	30	45/2	**15**	puruša	120	90
8	jānu	32	24	**16**	īšā	188	141

Remarks – (1) Many interrelations between the various units are calculated e.g. 1 aratni = 2 prâdeśa, 1 puruṣa = 4 prakramas = 30 angulas etc. (2) KSS defines same units except 1 vitasti = 12 angulas, 10 vitastis = 1 puruṣa. (3) MSS assumes 1 prâdeúa = 10 angulas, 1 vitasti = 12 angulas, 1 aratni = 2 vitastis. (4) Units like aratni, prâdeúa, vitasti, pada, prakrama, puruṣa etc are found used in Śatapatha Brâhmaṇâ for similar measurements.[16]

Bricks Used for Constructions -

25 different types of bricks used for constructing fire altars are elaborated in BSS and ASS along with their dimensions. Here these bricks are classified according to their shapes. Puruša equal 120 angulas is the standard unit of measurement. All the dimensions are noted in angulas. Section and dimensions of each brick are tabulated below.

No	Name	Section	Dimensions
		Square Shape Brick	
1	caturthī	(1/4) th part of puruša	30 × 30
2	pañcamī	(1/5) th part of puruša	24 × 24
3	šašṭī	(1/6) th part of puruša	20 × 20
4	daśamī	(1/10) th part of puruša	12 × 12
5	pādya of caturthī	(1/4) th part of caturthī	15 × 15
6	pādya of pañcamī	(1/4) th part of pañcamī	12 × 12
		Rectangular Bricks	
7	ardhya of caturthī	(1/2) part of caturthī	30 × 15
8	adhyardha of caturthī	(3/2) part of caturthī	45 × 30
9	ašṭamī of caturthī	(1/8) part of caturthī	15 × 15/2
10	adhyardha of pañcamī	(3/2) part of pañcamī	36 × 24
11	ašṭamī of pañcamī	(1/8) part of pañcamī	12 × 6
12	sapādya of pañcamī	1 & 1/4 part of pañcamī	30 × 24

Triangular Bricks			
13	ardhya of caturthī	(1/2) part of caturthī	30,30, 30?2
14	pādya of caturthī	(1/4) part of caturthī	30, 15 $\sqrt{2}$, 15 $\sqrt{2}$
15	ašṭamī of caturthī	(1/8) part of caturthī	15, 15, 15 $\sqrt{2}$
16	ardhya of pañcamī	(1/2) part of pañcamī	24, 24, 24 $\sqrt{2}$
17	pādya of pañcamī	(1/4) part of pañcamī	24, 12?2, 12 $\sqrt{2}$
18	ašṭamī of pañcamī	(1/8) part of pañcamī	12, 12, 12 $\sqrt{2}$
19	ardhya of adhyardha of pañcamī	(3/4) part of pañcamī	36, 24, 12 $\sqrt{13}$
20	dīrghapādya of adhyardha of pañcamī	larger part when rectangle is cut by diagonals	36, 6 $\sqrt{13}$, 6 $\sqrt{13}$
21	śulapādya of adhyardha of pañcamī	One of the smaller part when rectangle is cut by diagonals	24, 6 $\sqrt{13}$, 6 $\sqrt{13}$
22	ašṭamī of dīrghapādya	(1/2) part of brick No 20	18, 12, 6 $\sqrt{13}$
23	ašṭamī of śulapādya	(1/2) part of brick No 21	12, 18, 6 $\sqrt{13}$

24. Ubhai Brick – This brick is obtained by juxtaposing two bricks No. 22 and 18 along equal side 12. This results in a triangular brick with dimensions 30, 6 $\sqrt{13}$, and 12 $\sqrt{2}$.

25. Hamsamukhi Brick – Pentagonal brick with dimensions 30, 15/2, 15 $\sqrt{2}$, 15 $\sqrt{2}$, 15/2.

Vedis – Information on various Vedis contained in BSS [17] is collected here.

Name	**Shape**	**Dimensions**
āhavaniya	Square	Area: 9216 sq.angulas
gārhapatya	Square, Circle	Same area
Dakšināgni	Semi circle	Same area
Mahāvedī	Isosceles trapezium	24, 30, altitude 36, Area 972
Sautrāmaṇiki vedi	Isosceles trapezium	Area 324 sq.angulas
Paitṛki vedi	Isosceles trapezium	Area 108 sq.angulas
Paśubandha vedi	Isosceles trapezium	Area 108 sq.angulas
Prāgvamśa vedi	Rectangle	Area 192 or 120 sq.angulas

Citis – Fire Altars - BSS elaborates 14 types of fire altars, each having area seven and half square purusa and constructed using 200 bricks.

Name	**Shape**	**Reference**
caturasra śyenacit type I and II	Square falcon	BSS Ch. 8, Ch. 9
vakrapakśa śyenacit type I and II	Falcon shape with curved wings, outspread tail	BSS Ch. 10, Ch.11
kankacit	Kite shape body and tail of altar	BSS Ch. 12
alajacit	The body, the head and tail of altar in form of alaja bird.	BSS Ch.13

praugacit	An isosceles triangle.	BSS Ch. 14
ubhayatah prauga	Rhombus	BSS Ch. 15
rathacakracit	Chariot wheel	BSS Ch. 16
caturasra droṇacit	Square Through	BSS Ch. 17
parimaṇdala dronacit	Circular Through	BSS Ch. 18
śmaśānacit	Altar in form of pyre	BSS Ch. 19
vakrāñga kurmacit	Tortoise form with twisted or angular wings	BSS Ch. 20
primaṇđala kurmacit	Tortoise form with rounded or circular wings	BSS Ch. 21
samūhyacit	Circle. Mud is used in place of bricks	
chandaścit	Falcon shape. Only mantrās are recited while touching the places at which bricks are to be placed	BSS Ch. 7

Remarks – (1) ASS had discussion on rectilinear śyenacit of two types [Ch. 10, 11], Praugacit and ubhayatah praugacit [Ch. 12], rathacakracit and square droṇacit [Ch. 13], vakrapakśa śyenacit type I and II [Ch 15 - 20], and chandaúcit. (2) KSS describes the construction of droṇacit [Ch. 4], caturasra śyenacit type I and II [Ch 6, 13], and alaja and kankacit [Ch. 14]

Mathamatics in Śulvasûtras

From mere observation of the data regarding design and construction of the bricks, vedis and citis we can conclude that śulvakâras (Composer of Śulvasûtras) must have extensive knowledge of basic properties of plane figures, their areas, similarity relations and so called Pythagoras theorem etc. Here we first enlist some properties not explicitly mentioned but widely applied in the scripts. And later geometrical constructions for combination and transformation of areas will be noted.

Properties of Plane figures.

(1) A line segment can be divided into any number of equal parts. (2) A circle can be divided into any number of parts by drawing diameters. (3) The diagonal of the rectangle bisects it. (4) The diagonals of rectangle bisect each other. (5) The diagonals of rectangle divide it into four parts such that two opposite triangles are same in all respect. (6) The diagonals of rhombus / square bisect at right angles. (7) The triangle can be divided into number of equal and similar parts by dividing the sides into equal number of parts and joining the points two and two. (8) The isosceles triangle is divided into two equal parts by the line joining the vertex with middle point of the opposite sides. (9) A triangle formed by joining the extremities of any side of a square to the middle point of the opposite is equal to half the square. (10) A quadrilateral formed by the lines joining the middle points of the sided of the rectangle is a rhombus whose area is half that of rectangle. (11) A parallelogram and the rectangle which are on the same base and within same parallels are equal. (12) The maximum square that can be described within the circle has its corners on the circumference. (13) The perpendicular bisector of the line segment is locus of equidistant points from end points of

the segment. (14) The tangent of the circle is perpendicular to radius at the point of contact. (15) A quadrilateral formed by joining the mid points of square is itself a square with half the area of Original Square. (16) The corresponding sides of similar figures are proportional. (17) The area of similar triangle is proportional to square of their sides. [18]

Remarks: (1) This list is not exhaustive. (2) Some of the properties like No 1, 2, 8 14 are quite obvious. (3) Others can be verified by drawing the figures and with the help of elementary geometrical knowledge.

Geometric Constructions –

(A) The geometric constructions explicitly stated by Śulvasûtras are listed below.

(1) To draw a line perpendicular to given line or perpendicular bisector. [KSS 1.2]

(2) Construction of all types of triangles with given conditions. [ASS 1.2]

(3) To draw a rectangle and rhombus. [BSS 1.6, ASS 2.1, KSS 1.3]

(4) To construct a square with given side. [BSS 1.4 -6; ASS 1.2, 3, 7; KSS 1.3, MSS 1.11]

(5) To construct a trapezium of given altitude, base and face. [BSS 1.7, KSS 2.10 - 12]

(B) The constructions which can be inferred from other complex constructions are -

(1) To draw a straight line at right angles to a given line from a given point. [19]

(2) To construct a parallelogram having given sides at a given inclinations. [20]

(3) To construct a circle. (4) To divide a circle into number of parts by drawing diameters.

(5) To divide a line into number of equal parts. (6) To construct square or isosceles trapezium of given area. (7) To construct a isosceles trapezium similar to given isosceles trapezium but with one third or double its area [21]. (8) To construct fire altar similar to that of the shape of falcon, but differing from its primitive area of seven and half square purušas by m square purušas [22]

Combination of Areas –

Śulvakâras attempted to draw a square whose area is equal to area of given plane figures. Such constructions are listed below.

(1) To construct a square equivalent to n times a given square [23]. (2) To construct a square equivalent to n th part a given square [24]. (3) To construct a square equivalent to sum of two different squares [BSS 2.1, ASS 2.4, and KSS 2.13]. (4) To construct a square equivalent to difference of two different squares [BSS 2.2, ASS 2.4, and KSS 2.13]. (5) To construct a square equal in area of two given triangles [25]. (6) To construct a square equal in area of two given pentagons [26]. (7) Construction for combination of two different squares [BSS 2.12 and KSS 2.8].

Transformation of Areas –

Śulvasûtras elaborated transformations of given plane figure to desired plane figure having approximately same area. Such transformations are listed below.

(1) Transformation of triangle into rectangle or square [KSS 4.1, 2, 5]. (2) Transformation of rectangle or square into triangle [BSS 2.7, ASS 2.7, KSS 4.3]. (3) Transformation of rectangle into square [BSS 2.5, ASS 2.7, KSS 3.2]. (4) Transformation of square into rectangle [BSS 2.3, ASS 2.7, KSS 3.2]. (5) Transformation of rectangle or

square into trapezium [BSS 2.6, ASS 12.4, KSS 3.2]. (6) Transformation of trapezium into square or rectangle [ASS 5.7]. (7) Transformation of rectangle or square into rhombus [BSS 2.8, ASS 12.8, KSS 4.4]. (8) Transformation of rhombus into rectangle or square [ASS 5.7].

(9) Transformation of square into circle [BSS 2.9, ASS 3.2, KSS 3.11, MSS 1.8 (a)].

(10) Transformation of circle into square [BSS 2.10, ASS 3.3, KSS 3.11, MSS 1.8 (b)].

The Śulva Theorem –

The statement and its application of the theorem commonly known as the Pythagoras theorem is elaborated by all the four úulvasûtras [27]. BSS states that "*dîrghacaturaúrasyâkòayarajjuh pârúvamânî tiyañmanî ca yatpºthagbûte kutastadubhayam karoti*" i.e. The diagonal of a rectangle produces both areas which its length and breadth produces separately" [BSS 1.3] Thus in rectangle ABCD, $AC^2 = AB^2 + BC^2$. The similar statement is given by ASS [1.4], KSS [2.7] and MSS [10.10]. As an example úulvakâras mentioned following integer, rational and irrational triples at various places followed or preceded by the general statement. These triples are – 3, 4, 5; 7, 24, 25; 15, 8, 17; 12, 5, 13; 12, 35, 37; and 15, 36, 39 [BSS 1.5]; 20, 15, 25 ASS [5.3, 1.2,]; 16, 12, 20; 12, 9, 15; [ASS 5.3]; 6, 5/2, 13/2; 5, 25/12, 65/12; 10, 25/6, 65/6 [ASS 6.6,7,8]; 188, 235/3, 611/3 [ASS 6.3]; 27, 45/4, 117/4; 18, 15/2, 39/2 [ASS 7.1, 2] 1, 5/12, 13/12 [KSS 1.4]; 1, 3, "10; 2, 6, "40 [KSS 2.4, 5]; 1, "2, "3 [KSS 2.10]; 40, 96, 104; 188, 52, 194; [MSS 1.4-6]; 6, 9/2, 15/2; 1 "10, "11 [MSS 2.5];

Remarks: (1) Some rational triples are submultiples of integer triples. (2) The converse of the theorem, though nor explicitly stated in scripts but applied many times.

Geometric Algebra –

We note some of instances from Śulvasûtras where geometrical constructional problems demands algebraic solutions. B Datta remarked that "The geometrical constructions described in (Śulvasûtras) are of considerable algebraic significance. They indeed form the seeds of Hindu geometric algebra"[28].

(1) Enlargement of isosceles trapezium leads to equation $972 x^2 = 972 + m$.

(2) Enlargement of falcon shaped altar first type demands solution of $x^2 = 1 + (2 m / 15)$.

(3) Enlargement of falcon shaped altar second type leads to solution of $7 x^2 + (1/2) x = (7/2) + m$.

(4) For aśvamedha Vedi we need solution of $7 x^2 + (1/2) x = 43 / 2$.

(5) Kâtyâyana's formula for rational triangles generates second degree equation $x^2 + y^2 = z^2$ with solution

$m^2 + [(m^2 - 1)/ 2]^2 = [(m^2 +1) / 2]^2$ [29].

(6) Simultaneous indeterminate equations have some footing in Śulvasûtras with respect to altar construction. (A) For Gârhapatya agni one need to solve the equations $x + y = 21$,

$(x / p^2) + (y / q^2) = 1$, where number of bricks used are 21 and size of bricks are 1/p, 1/q.

Baudhayana's solutions are x = 16, y = 5 when p = 6, q = 3 and x = 9, y = 12 when p = 6, q = 4. (B) In case of falcon shaped altar with 200 bricks and 7and half sq. puruṣas Baudhâyana uses four verities and solve the equations $x + y + z + w = 200$,

$(x/p) + (y/q) + (z/r) + (w/s) = 15/2$. The solutions are x = 24, y = 120, z = 36, w = 20 when p = 16, q = 25, r = 36, s = 100. (C) Âpastamba uses five verities for the same problem and one need to solve $x + y + z + u + v = 200$, $(x/p) + (y/q) + (z/r) + (u/s) + (v/t) = 15/2$ and provide the solutions x = 67, y = 58, z = 48, u = 18 and v = 9 when p = 16, q = 25, z = 48, r = 64, s = 100 and t = 144. [30]

Surds in Śulvasûtras –

From the dimensions of Vedis Cities and bricks mentioned earlier it is clear that śulvakâras used values of $\sqrt{2}$, $\sqrt{3}$ etc and knew the basic operations on surds. Baudhâyana defines the value of $\sqrt{2}$ as

$\sqrt{2}$ = [1 + (1/3) + 1/(3×4) – 1/ (3×4×34)] = 577 / 408 [BSS 2.12, ASS 1.6] = 1.4142157

When compared to current value 1.414213—— above value is correct up to 5 decimal places.

Value of Pi Ratio –

The pi ratio is not explicitly mentioned but can be inferred from constructions like transformation of circle to square and vice versa. The inferred values are in the range of 3 to 3.2. Gupta R.C. mentioned 15 possible values π [31] and remarked that best possible value of π their in is the approximation based on interpretation of verse 11.15 of MSS. This value is ð = 25/8 = 3.125.

This author is pleased to humbly note that this account of ancient Indian scripts 'Śulvasûtras' is capable of giving fair idea of progress of science of geometry in very early period of the subject. *Ityalam*.

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[15] ibid p 114.

[16] Satya Prakash [4] above Chapter 3.

[17] Sen [3] above, Chapter 3, 4, and 7.

[18] SOS [2] above, Chapter IV, p 41 – 51.
[19] SOS [2] above, p 53.
[20] SOS [2] above, p. 67
[21] SOS [2] above, p. 153.
[22] SOS [2] above, p. 154.
[23] SOS [2] above, p. 71.
[24] SOS [2] above, p. 74.
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[26] SOS [2] above, p. 80
[27] SOS [2] above, p. Chapter IX
[28] SOS [2] above, p. 165
[29] SOS [2] above, p. 178.
[30] SEN [3] above, p. 183 – 186.
[31] Gupta R.C.: New Indian Values of π from the Mânava Śulba Sûtra, Centanrus (Denmark) 31, 1988, p 114-125.

Transliterated Verses / Important Words - (As they appear in the article)

अस्यमहतो भूतस्य निश्वसितमेतद् यद ऋग्वेदो यजुर्वेदः सामवेदः अथर्वंगिरसः ।
कल्पोनाम विधिः ।
कल्पन्ते समर्थ्यन्ते यज्ञ यागादि प्रयोगः यत्र स कल्पः ।
स्वल्पाक्षरं असन्दिग्धम् सारवद् विश्वतोमुखम्
अस्तोभम् अनवध्यम् च सूत्रम् सूत्रविदो विदुः ।।

Words:

वेदांग, कल्प, संहित, श्रौतसूत्र, बौधायन, आपस्तंब कात्यायन, मानव, वेदी, चीती, अंगुल, पुरुष, चतुर्थी, पंचमी, अर्ध्य , अध्यार्ध, आहवनीय, गार्हपत्य, महावेदी, श्येन, वक्रपक्ष, कंक, अलज.

Geometry in Tantra

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FOREWORD

The word “Tanthra” has several meanings such as – a loom, a thread, a ritual, the principal doctrin etc etc. But in the context of subjects, Tanthra or “Tanthra saastra” specifically refers to the literature related to a special system of religious practices, which form a branch of the ancient Hindu tradition and which are discending down from the pre historic periods. There are several schools of thought on the age of the ancient Tanthra Text books and some scholars prefer to place them at an age prior to that of the Vedas. However, a majority of the traditional scholars prefer to place them immediately after Vedas and before Puranas. The age of Veda is a subject of continuous research and the most recent researches on their internal evidences takes the Vedas to around 6 to 8 thousand BC.

Even modern western scholars like Sir **John Woodroffe,** who studied Tanthra literature extensively prefer to take them to a very antiquate period.

There are many difficulties in determining the dates of each text book of Tanthra and the present article does not make an attempt for that. What ever may be the controversies about the time of the Tanthra text books, one important fact which we can not miss is that the roots of Tanthra are clearly seen in the Vedas.

Surprisingly, even the mathematical complexities of the Tanthra tradition, have their clear roots in the Veda. Unfortunately, the tradition of the Tanthra discipline is scattered and a mighty treasure of scientific knowledge, stored in that literature, is at the verge of extinction. The scholars who can interpret and explain the scientific aspects of the Tanthra are very rare, now a days.

The author starts his exploration of Tanthra directly from the Veda and proceeds to the earliest Tanthra texts and then covers some of the medieval texts as well. Even though he prefers not to mention the period of the texts concerned, we have to keep the above points in mind and appreciate the marvelous mathematical acumen of the antiquity.

-Editor

पूर्णमदः पूर्णमिदम् पुर्णात् पूर्ण मुदच्यते
पूर्णस्य पुर्णमादाय पूर्णमेवा वशिष्यते

pūrṇamadah pūrṇamidam purṇāt pūrṇa mudacyate
pūrṇasya purṇamādāya pūrṇamevāvaśiṣyate

The unseen universe is infinite. Universe that which is seen is also infinite. The visible universe came out of that previous primordial universe. Even after the outcome of this universe, the original universe remains infinite.

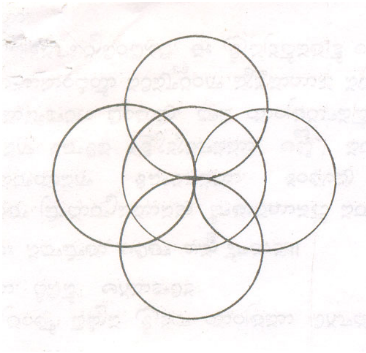


Fig1

The seen infinite universe comes out of the primordial infinite because of its internal creative activity. Even then the primordial does not undergo any change and it still remains infinite. In general when an active work is done some energy is lost. But when the energy is infinite such loss does not happen. Hence it remains infinite.

Such infinite is represented by a (Bindu) circle in planar geometry. Here infinite energised space is enclosed in a circle. The same fact is expressed by the following Vedic sentence.

सहस्र शीर्षा पुरुषः सहस्राक्षः सहस्रपात्
सभूमिम विश्वतोवृत्वा अत्यतिष्ठ दृशाञ्गुळम्।।
sahasra śīrṣā puruṣaḥ sahasrākṣaḥ sahasrapāt
sabhūmima viśvatōvūtvā atyatiṣṭha ddaśāñguḷam..

Every point on the circumference of a circle can be treated as its (शीर्ष)vertex. Thus the circumference of a circle is a set of infinite points. This is the sahasra śīrṣā property of circle.

The diameter of circle can be called as its axis. The axis of symmetry of the circle is any of its diameter is its सहस्राक्षत्व (sahasraaksatva) A circle can be considered as an infinite sided polygon. This is its सहस्रपादत्व (sahasrapādatva)

Thus an infinite circle has encircled the entire universe. Its circumference is said to be ten units.
Utpalāchārya, an Indian mathematician has taken the value of π as $\sqrt{10}$. Also

The western mathematician, Plolemy has taken $\pi = \sqrt{10}$. In this context

$\therefore$ Circumference of a circle = π D = 10

When $\pi = \sqrt{10}$, then D $\sqrt{10}$ = 10 $\Rightarrow$ D = $\sqrt{10}$

Hence the diameter of a circle is the square root of the length of the circumference.
The evolution of the seen universal circle from the infinite circle is explained in Vedic stanzās below.

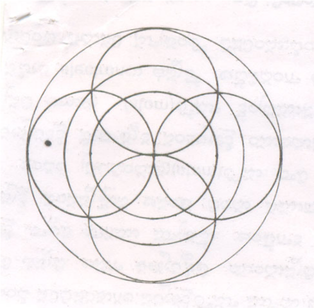
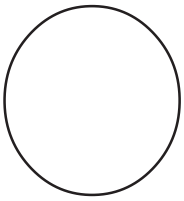


Fig2

सजातो अत्यरिच्यत पश्चाद्भूमि मधोपुरः।।

sajātoo atyaricyata paśc̄ādbhūmi madhōpuraḥ

From the primordial (Mahaa Bindu) infinite circle four infinite circles evolved. But the centers of these circles lie on the original primary circle. They are of (Sajaatiya) the same diameter. The Vedic stanza explains geometrically the formation of four circles. They are on both sides of the centre, one above and one below.



Thus the creation is mentioned in the four of five circles. One shall notice that this is the manifestation of the universe as mentioned in Vedic lines.

श्री कला यंत्रम् (śrī kalā yantram)

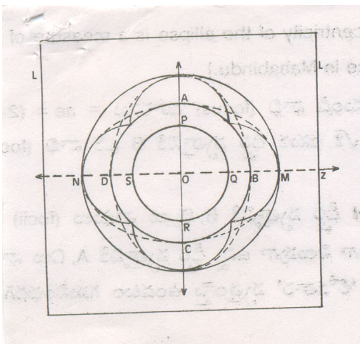


Fig-3

अखन्ड मन्डलाकारम् विश्व व्याप्य व्यवस्थितम्
त्रैलोक्यमन्डितम् येन मन्डलम् तत् सदा शिवम्
akhanḍa manḍalākāram viśva vyāpya vyavasthitam
trailōkyamanḍitam yēna manḍalam tat sadā śivam - meru tantra

The above said five circled creation is enclosed in an infinite circle called Sadaasiva circle. Let another circle be drawn through the points of intersection of the four circles. So obtained seven circle geometrical figure is stated by the Vedic stanza.

सप्तास्यासन् परिधयः त्रिस्सप्त समिधः कृताः
saptäsyäsan paridayaha trisaptha samidhah krutäh - veda

Finally the figures contains seven circles and twenty one prominent (Samidha) points.

On outermost
sadāśiva circle the number of points of contact = 4
On second circle the number of points of trisection = 4
On the internal circle the number of points of intersection = 8
The number of centres of the circles = 5
In all the number of prominent (**Samidha**) points in that seven circled Yantra is (**Trisapta** =3x7) = 21

The same Vedic sentence gives one interesting information when **Katapaya formula** of Vararuchi is applied.

1	2	3	4	5	6	7	8	9	0
क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
Ka	kha	ga	gha	inya	cha	chha	Ja	Jha	Ini
ट	ठ	ड	ढ	ण	त	थ	द	ध	न
Ta	Tah	da	dah	Ana	Tha	Tha	Da	Dha	Na
प	फ	ब	भ	म					
Pa	Pha	Ba	Bha	Ma					
य	र	ल	व	श	ष	स	ह	क्ष	
Ya	Ra	La	Va	Sa	Sha	Sa	Ha	Ksha	

Let the same Vedic sentence be interpreted with the help of above (Sankhya - Maatruka)
Letter – number relation.

सप्तास्यासन् परिधय: त्रिस्सप्त समिध: क्रुता:
saptāsyāsan paridhayaha trissaptha samidhah krutāh

परिधय: Paridhayaha = circumference of circles of

सप्ता, आस्य (saptā āsya) = 7 units diameter

क्रुता: Krutaah = becomes

त्रिस्सप्त Trisapta = 21 Sa, = 7 = (Ma+e = 5 + 3) = 8 , ध = 9

Samidha = 987 (Ankaanaam Vaamatogatih)

Thus 21 is the integral part and 987 is the decimal part of the said number.

$\therefore$ Trissapta Samidhah = 21. 987

$\therefore$ Circumference = $\pi D = \pi(7) = 21.987$

$\Rightarrow \pi = 21.987/7 = 3.141$(an approximation to 22/7)

Thus the value of the universal constant π is suggested in this Vedic sentence. This is the secret meaning of the Vedic sentence.

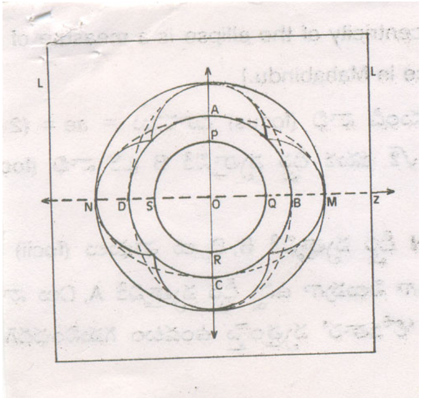


Fig:3

P , Q , R , S are the four centres of the four small circles.
Let the radius of the circle PQRS = r

Radius of the circle ABCD = $r\sqrt{2}$

Radius of the external circle = 2r

$\Rightarrow$ OQ = r , OB = $r\sqrt{2}$, OM = 2r

As seen in the above figures if an ellipse is drawn through the points A , M , C , N then its Semi-major axis = OM = 2r = a

Semi-minor axis = OA = $r\sqrt{2}$ = b

Equation of the ellipse is $(x^2/a^2) + (y^2/b^2) = 1$

$\Rightarrow (X^2/4r^2) + (y^2/2r^2) = 1$

Eccentricity of the ellipse = e = " $\sqrt{(a^2 - b^2)/a^2}$ = " $\sqrt{[(4r^2 - 2r^2)/4r^2]} = 1/\sqrt{2}$

The eccentricity of the ellipse is a measure of the intensity of disturbance in Maha Bindu.

Distance of the focus from the centre = ae = (2r) $(1/\sqrt{2}) = r\sqrt{2}$

$\Rightarrow$ OB = $r\sqrt{2}$

Hence B is a focus of the ellipse. Similarly D is the other focus.

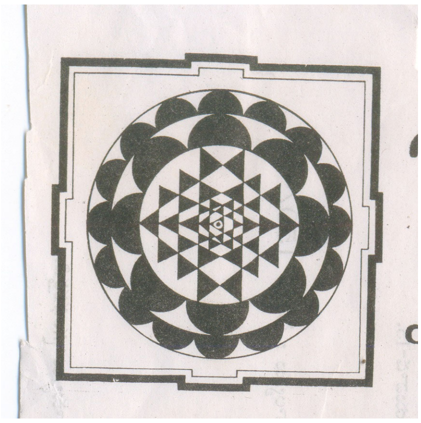
Hence B and D are the two foci of the elleipse AMCN

In similar way A, C become the foci of the vertically sen ellipse.

Clearly the four foci A, B, C, D lie on the middle circle of Srikala yanthra. The distance of the directrix of the ellipse from the centre = oz = a/e = 2r /(1/$\sqrt{2}$) = $2r\sqrt{2}$.

Thus the two lines L_1 and L_2 are the two directrices of the ellipse AMCN Similarly the two lines K_1 and K_2 are the two horizontal directrices of the vertical ellipse. In all the four directrices from a square. This square is called the (भूपुरम्) Bhupuram (सदाशिव)of the yantra. Also the outer / circle is the auxilary circle for the two ellipses.

SRI CHAKRAM



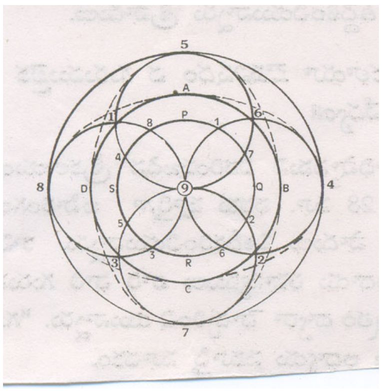
बिन्दुत्रिकोणकाश्टावतार युगलोकपत्रव्रुत्तयुतम्
वसुदळ व्रुत्त कलादळव्रुत्त त्रिमहिग्रुहम्भजे चक्रम् – विद्यारण्य
bindutrikoṇakāśṭāvatāra yugalokapatravruttayutam
vasudaḻa vrutta kalādaḻavrutta trimahigruhambhaje cakram - vidyāraṇya

From the center, the sequence of the Avaranaas is described by the Stanza. Central point, then the triangles one, eight ten fourteen are there in order. Then eight petals, sixteen petals and then the three parallel lines called the ‘Bhupura Trayam’.

षण्णवत्युन्गुळायामम् सूत्रम्प्राग्प्रत्यगायतम्
चतुर्भिरन्गळैः सिश्टैः सम्व्रुतानि च भूपुरम्
अन्तर्नवान्गुळम्ग्नेयम् मध्ये पत्रन्तु षोडशः
एकदशाञ्गुलम्ज्ञेयम् अष्टपत्रम् समालिखेत् (प्रपंच सार संग्रहम्)

śaṇṇavatyunguḻāyāmam sūtramprāgpratyagāyatam
caturbhirangḻaihi siśṭaihi samvrutāni ca bhūpuram
antarnavānguḻamgneyam madhye patrantu ṣōḍaśaḥ
ekadasāngulamgneyam aṣṭapatram samālikhet (prapañca sāra saṅgraham)

From East to West take a length of 96 units. Four units from East occupy Bhupura
Trayam, 16 petaled space is 9 units and eight petaled space is 11 units. Hence both sides put together the Bhupura and the petals space occupy **2(4+9+11) = 48** units. Hence the remaining 48 units is the diameter of the internal circle.

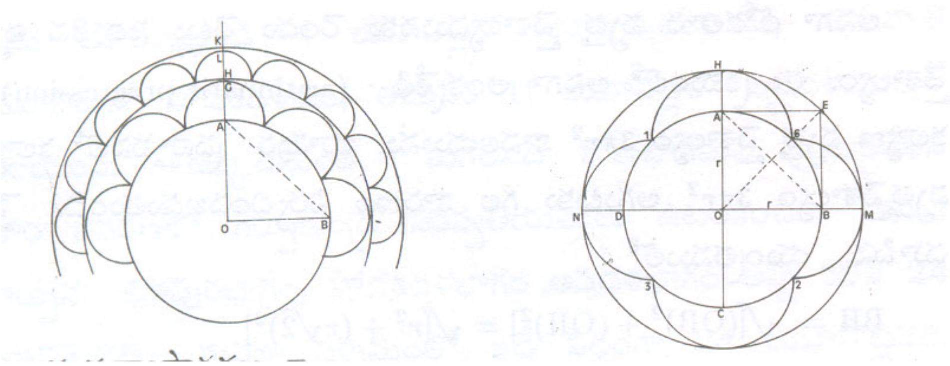


The principle of the ellipse and its directrix can be applied in the study of Sri Chakram. Consider the internal circle as the circle on the minor axis as diameter of an ellipse. The foci lie on the same circle. Then its eccentricity

$e=\mathbf{1}/\sqrt{2}$.

Let the radius of the internal circle= r = ae. Then the semi major axis **OM = = r** $\sqrt{2}$ **a**. Distance **OZ = 2r= a/e** Where **r = 24 units** then **OZ=48 units**.

⇒ Distance between the two lines L_1 and L_2 is 96 units. Thus the outer Bhupura lines are the directrices of the ellipse. Thus the four outer Bhupur lines indicate the presence of two vertically intersecting imaginary ellipses. These ellipses determine the real placement of the second and the circles of Srichakra. The second circle becomes the auxilary circle to the ellipse.



On this internal circle eight petals(Ashta dala) are constructed so that the center of each semi-circle lies on the internal circle. Then the radius of each (Ashta dala) petal =2r Sin(π /16)= then the distance between the auxillary circle and a petal of Ashta dala =OH-(OA+AG)=r $\sqrt{2}$ -[r+2r Sin(π /16)] = 0.024r.

This difference is used to determine the height of the circle in three dimensioned Meru Prasthara Sri Chakram.
In the same way the outer circle will be the director circle for the said ellipse. The radius of the director circle = $\sqrt{a^2+b^2}$ $\sqrt{(r\sqrt{2})^2+r^2}=\sqrt{3}r^2=r\sqrt{3}$

Sixteen Petals(semi circles) are constructed on the auxillary circle with their centres on the auxillary circle. The radius(of semi circle) of each petal of sixteen petals**=2r** $\sqrt{2}$ **Sin(** π **/32).** Now the distance between the petal and the director circle **=r** $\sqrt{3}$ **-[r** $\sqrt{2}$ **+2r** $\sqrt{2}$ **Sin(** π **/32)]=0.04r**. This determines the height of the outer circle in three dimensional Meru Prasthara SriChakra.

The internal circle has a diameter of 48 units. To draw the triangles part in it, the division of the diameter is given as below by Tanthra sentence.

स्तुतो मे गन्गावल्ली स्तुतेति

- तन्त्रम्

stuto mey gangāvallī stuteti

- tantram

Applying "vararuchi rule", the letters indicate the numbers given below:

स्तु stu = 6, तो to =6 मे mey = 5, गम gam = 3 , गा gā = 3 व va = 4

ल्ली llī = 3 स्तु stu = 6 ते te = 6 ति ti = 6

The total of these numbers=48=diameter.The method of drawing the lines and the construction of the triangles is given in detail in Tantra texts. In Srichakra upasana the nine Avaranas are attributed with nine letters of the alphabet. They are called the Prakrutis of that Avarana plane.

TABLE

Avaranam	bindu	Δ	8 Δ^8	10 Δ^8	10 Δ^8	14 Δ^8	16 Petals	8 Petals	Bhupur
Alphabet	0	ma	ka	ra	yey	ee	ha	sa	la

From Bhupuram to the central Bindu the nine Avaranas are attributed with nine (Manthra Bijas) alphabet in Upasana Tantra.

TABLE

(1) Avarana	(2) No of Avaranas	(3) Bija	(4) Equivalent	(2) x (4)
Bhupuram	10	La	3	30
Shodasara	16	Sa	7	112
Astadala	8	Ha	8	64
Manvasra	14	Ee	4	56
Inner dasara	10	Ye	6	60
Outer dasara	10	Ra	2	20
Ashtakona	8	Ka	1	8
Trikona	3	Ma	5	15
Bindu	0	om	9	0
				365

In the first column the Avarana names are given. In second column the number of places in that Avarana is given. In third column the Manthra Bija letter is shown. In fourth column the number equivalent to the Bija letter is given as per to Vararuchi's Number-Alphabet rule. In the last column the product of columns (2) and (4) is calculated. It is astonishing to observe that the total of the last column comes out to be **365.** This number 365 reckons to number of days of a solar cycle. In Tantra, it is mentioned as Samvathsara Prajapathi. It is an accidental verification.

In the above table the Bija number (column (4)) and corresponding number of places of the corresponding Avarana (column-2) are taken alternately to form a continued fraction given below

3+1/7+1/16+1/8+1/8+1/4+1/14+1/6+1/2+1/10+1/1+1/8

3+1/7+1/16+1/8+1/8+1/4+1/14+1/6+1/2+1/10+1/1+1/8+1/5+1/3

=3.1416024

The value of this continued fraction to seven decimal places is an approximation to the value of **π.** This value 3.1416 is determined to be the value of π by the ancient mathematician ***ARYABHATTA.*** Let the continued fraction be taken to three numbers.

3+1/ (7+1/16) = 355/113 = 3.14159292

This number (355/113) is taken on the value of π by Chinese**.**

RARE MAGIC SQUARE

Numerals are very well used in Tantra. Magic squares of Bhuvaneswari Kachchaputi Tantra are famous in Tantra sastra. There is one 8 x 8 magic square in Tantra whose author and his time is unknown. The extract of that magic square written in notation form is given below.

पंचविंशति मारभ्य अष्टाशीत्यंतिमो यथा ।
चतुःषष्टि प्रकोष्टेषु लिखेत् कटपयादिन ।।

pañcaviṃśati mārabhya aṣṭāśītcantimō yathā .
catuḥṣaṣṭi prakōṣṭēṣu likhēt kaṭapayādina ..

तेषां स्थानानि संप्रोक्तम् तिर्यगूर्ध विधानतः ।
वर्णद्वंद्व पदैर्युक्तं तत्तत् कोष्टस्थिति क्रमम् ।।

tēṣāṃ sthānāni samprōktam tiryagūrdha vidhānataḥ .
varṇadvandva padairyuktaṃ tattat kōṣṭasthiti kramam ..

गया तोषं मखा वाणी यवो हंस सगोरजः ।
वेदं शीलं तथा लाभं रमा सारं हितोपयः ।।

gayā tōṣaṃ makhā vāṇī yavō haṃsa sagōrajaḥ .
vēdaṃ śīlaṃ tathā lābhaṃ ramā sāraṃ hitōpayaḥ ..

हेला पदं रवं साथो तारागणं वयोमषी ।
सीता रायोपणंदारं मासं भावं गदा तलं ।।

hēlā padaṃ ravaṃ sāthō tārāgaṇaṃ vayōmaṣī .
sītā rāyōpaṇandāraṃ māsaṃ bhāvaṃ gadā talaṃ ..

रसं छावं देहं कालं वेषं माया तमो गरम् ।
यारं दमं छयं रोषो बालस्तेजो शिवोवसुः ।।

rasaṃ chāvaṃ dēhaṃ kālaṃ vēṣaṃ māyā tamō gam .
yāraṃ damaṃ chayaṃ rōṣō bālastējō śivōvasuḥ ..

शमी भारं लता तापो सदा रागं पथोजवः ।
ताभं गाथा भगोमोहं हेयं पोषं खरोसमः ।।

śamī bhāraṃ latā tāpō sadā rāgaṃ pathōjavaḥ .
tābhaṃ gāthā bhagōmōhaṃ hēyaṃ pōṣaṃ kharōsamaḥ ..

एतत् च्छिवपदं प्रोक्तं तंत्रागम विषारदैः ।
पूरयेदुक्त मार्गेण मंत्र वर्णान् यथा क्रमम् ।।
साधयेदिष्ट काम्यानि मंत्र पूजा विधानतः ।।

ētat cchivapadaṃ prōktaṃ tantrāgama viṣāradaiḥ .
pūrayēdukta mārgēṇa mantra varṇān yathā kramaṃ ..
sādhayēdiṣṭa kāmyāni mantra pūjā vidhānataḥ ..

ga yā too śam

The above said Tantra stanzas explain the placement of the integers starting from 25 and upto 88 in 8x8 magic square g ग = 3, या yaa = 1. Thus the word "gayaa" indicate that the integer 25 be places in 3rd row- 1st column

तो to = 6 , शम् sham = 6

The word "tosham" indicates the place of 26 as 6th row – 6th columns of the square. Thus deciphering all the places of the numbers an astonishing 8x 8 magic square has formed.

40	65	60	29	51	86	79	42
50	87	78	43	37	68	57	32
25	64	69	36	46	75	82	55
47	74	83	54	28	61	72	33
62	27	34	71	73	48	53	84
76	45	56	81	63	26	35	70
67	38	31	58	88	49	44	77
85	52	41	80	66	39	30	59

Here the additional total of the numbers in any row or any column is equal to 452. Also the total of the numbers in each diagonal is equal to 452. Thus this is an 8 x 8 magic square given in tantra.

Another 8 x 8 magic square is defined by the following stanzas in the same Tantra.

शिवापदं ततो वक्ष्ये साधकाभीष्ट सिद्धये ।
येये संख्याः समुदिष्टाः पूर्वे कोष्ठ प्रपूरणे।।

śivāpadaṃ tatō vakṣyē sādhakābhīṣṭa siddhayē .
yēyē saṅkhyāḥ samudiṣṭāḥ pūrvē kōṣṭha prapūraṇē..

एतासां वर्ग संख्याभिः कोष्ठान्यत्रैव पूरयेत्
यदागच्छति तद्विद्दि महनीयं शिवापदं।।

ētāsāṃ varga saṅkhyābhiḥ kōṣṭhānyatraiva pūrayēt
yadāgacchati tadviddi mahanīyaṃ śivāpadaṃ..

कोष्ठाष्ट संख्या क्रम योजनाद्यै ।
नागांग नेत्रोरग दृक्भवेद्धि ।।

kōṣṭhāṣṭa saṅkhyā krama yōjanādyai .
nāgāṅga nētrōraga dṛkbhavēddhi ..

तंत्रोक्त मार्गेण नियोज्ययंत्रे ।
यजन्तु देवीं सकलेष्ट सिद्ध्यैः।।

tantrōkta mārgēṇa niyōjyayantrē .
yajantu dēvīṃ sakalēṣṭa siddhyaiḥ..

In the previously explained 8 x 8 magic square , each number of the block is to be replaced by the square of that number. Thus all the 64 block numbers may be replaced by the numbers obtained by squaring the existing numbers. Thus obtained 8 x 8 square matrix is also magic square.

1600	4225	3600	841	2601	7396	6241	1764
2500	7569	6084	1849	1369	4624	3249	1024
625	4096	4761	1296	2116	5625	6724	3025
2209	5476	6889	2916	784	3721	5184	1089
3844	729	1156	5041	5329	2304	2809	7056
5776	2025	3136	6561	3969	676	1225	4900
4489	1444	961	3364	7744	2401	1936	5929
7225	2704	1681	6400	4356	1521	900	3481

The total of the numbers of any row or column or the diagonal is given by

नाग näga = 8 , अन्गा angä = 6 नेत्रा neträ = 2

उरगा uragä = 8 , द्रुक् druk = 2

By mathematical rule this number is to be taken as 28268.

Thus this magic square of 8 x 8 matrix is a rare one. Till today no mathematical formula is found to arrive at the table 2 from table 1with same law of addition. This shows the divine intelligence of Indian Mathematician.

Maharishi Pingala Chandasutram and Computer Binary Algorithms

A Focus on reigniting Bharatiya thought on Binary system

(Sri Rajendran Mariappan, Vidya Bharati Dakshina Kshetra Veda Ganita Pramukh, Vidya Bharati Akila Bharatiya Shikshan Sansthan)

'Maharishi Pingala Chandasutram and Computer Binary Algorithms' is an unusual topic which links the past and present. Computers represent the modern era, the Vedas are of a hoary past. Much has been researched and documented about computers, the Vedas are still to be solved of their mysteries. Many Vedic hymns have astounded the modern scientists and astronomers, but there has been no serious effort to unravel the real meanings behind all the Vedic hymns. Here we present the relevant binary system sutras with the explanation and working of the algorithms written in coded sutras. This opens up a new areas for research and implementation of Pingala's left to right binary or the Big-endian system.

Of the various gifts the hindus gave the world, the knowledge of ganit (mathematics) is supreme. They gave the concept of Sunya (zero), the decimal system (base 10), sexadecimal (base 60) system. The Binary system, which forms the basis of computation and calculation in computers seems to be the superlative discovery of modern mathematics. It is astonishing to find the Binary system in the Vedanga of Chandas so clearly given by Maharishi Maharishi Pingala. As with any ancient Vedic knowledge, the Binary system has been hidden in the Chandasutram. The hindus unique method is of using Sanskrit aksharas (alphabets) for writing numbers left to right, with the place value increasing to the right. These are read in the reverse order from right to left – 'Ankanam vamato gatih'. The binary numbers are also written in the same manner as decimal numbers and read from right to left. We present the relevant sutras from Maharishi Pingala's Chandasutram. The algorithms are written as sutras. The algorithms are recursive in nature, a very high concept in modern computer programming language. We fix the date of this Vedanga based on the date of the Vedas. The life period of Maharishi is thus dependent on the period of the Vedas. For this we take the internal evidence of astronomy in the Vedas to get its date.

The very word Veda has a derivational meaning – fountain-head and illimitable store house of knowledge. Vedas are four in number Rig, Yajur, Sama, Atharvana. Veda mantras have been used as databases to record astronomical events, constants, and mathematical procedures. Very large numbers have been encoded using the algebraic code of Maharishi Pingala Chandasutram. The conformity between decimal and binary number is given in Adhvayoga. This has to be properly understood, these akshara binary numbers are not only used for enumeration and classification of chandas. Chanda means covering, hiding or concealing according to Vedic etimology. According to Panini it means Vedas and vedic language. Prastara gives the algorithm for changing an ordinal number to guru-laghu binary syllabic encoding. Similar is the scheme of Katapayadi changing numbers to meaningful mnemonics. (We have developed software programs of the algorithms given in Maharishi Pingala Chandasutram). The algorithms should have been formulated before the specific

Veda mantras. And Vedanga Jyotisham gives the algorithms for astronomical calculations. To memorise the large volume of astronomical data and calculation tables Maharishi Pingala binary system was used. This astronomical calculations were necessary for making rituals at appropriate time as given in the kalpa sutras. Maharishi Pingala defined two series of numbers, index or serial number and a quantitative. The quantitative series lists the meteric variations, and index number gives decimal values of the variations as per adhvayoga algorithm. This has been wrongly understood by other writers. The main purpose of Chandasutram is to give rules based on bija-ganit (algebra) for encoding the ganas or akshara combinations.

The Vedangas are for the proper study and interpretation of the Vedas. The six vedangas are 1. Shiksha (phonetics), 2. Kalpa (ritual), 3. Vyakarana (grammar), 4. Nirukta (etymology), 5. Chandas (meter), 6. Jyotisha (astronomy).

Of these Chandas is for the study of Vedic meter. This gives the very important algebraic Vedic Science of encoding and metrics. This is the Patha (foot) of the Vedas. This gives the cryptic astronomical, algebraical, geometrical and the method of Vedic interpretation. This has been in use in Tamil language grammar Tholkappiam. The science of metrics in Tamil is named as Yappilakanam. Almost all the technical terms of Chandasutram have similar word meaning in Tamil.

Interpretation of Vedas based on the encoding methods using Chandasutram gives a method of chanting supercomputer. The mantras are based on sound and not on written scripts. The duration of pronunciation, the rules for when a laghu (short vowel) is to be pronounced as guru (long vowel) gives the superiority of sound over script. And this forms the basis of committing to memory large numbers of astronomy using the coding schemes of Chandas.

Vedas are in different Chandas (meters). One meaning of Chandas is that it is knowledge which is to be guarded in secret and propagated with care. The Vedas are also described as chandas. The whole of Sama Veda consists of chandas. There is word in Tamil referring tamil language as Chandahtamil. Of the six vedangas Chandashastra forms a part essential to understand the Vedas. These Chandas have been studied in great detail.

The sutras are a wonder of Sanskrit literature. There is no other language in this world which has anything like the Sutras. Many technicalities are used throughout these sutras. In translations from a rich and comprehensive language to a poor and ill-equipped tongue, it is extremely difficult to bring out the force, the dignity, the sweetness, the majesty and the flow of the original language. The translators, however learned they may be and however brilliant their intelligence may be, have to remember the great gulf which separates their intelligence with that of the great Maharishi Pingala who is their original author.

At best the translator can only explain and illustrate what he understands and conceives to be the meaning of the original author. It may be the correct interpretation of the author or what may have

been understood to be the meaning by the translator. In the case of the works of the Maharishis, I may not be very wrong if I say that none of the commentators or the translators could ever hope to come up to their standard. However, a man can do at best what he honestly knows to be the meaning; and I can assure that in the translation of these difficult sutras I have taken the greatest care to bring out the correct meaning of Maharishi Pingala in his inimitable work Chandasastra.

Everyone cannot compose a sutra. A sutra, to be a sound one, must have certain characteristic features and unless these conditions are satisfied they cannot pass muster under the heading of a sutra. In English, so far as my humble knowledge and practical experience go, it looks impossible to frame a sutra as the Maharishis have composed and conceived it. The defects of languages cannot be set right by ordinary men, and it is hopeless to make such attempts.

Maharishi Pingala, through his great love for the people, framed these sutras and they have to be interpreted on certain principles which the Maharishis have laid down for our guidance. The brevity of a sutra is its distinguishing feature, and it can easily be committed to memory even by the ordinary students. Somehow or other, Sanskrit language seems to have a close affinity to strengthen and improve memory.

Have we ever seen a Professor or Lecturer who is able to repeat a few sections of any book of his branch of learning? Is there any English knowing person who can repeat a play of Shakespeare or a few pages of any dictionary?

In Sanskrit, Dasopanishads are easily committed to memory. There are Dwivedis and Thrivedis who easily commit to memory one, two and three Vedas and repeat them with an ease which surprises the listeners. There are Chaturvedis who have commited to memory all the the four Vedas. There are many who have committed to memory the Sanskrit lexicon “Amara” and quote its stanzas offhand with the greatest ease. Bhagavata, Ramayana, Maha Bharata and other extensive works are easily committed to memory. Kavyas and Natakas. Epics and Dramas form no exception. Astrological literature, whether it be stronomical calculation or astrological Phalabhaga has been committed to memory.

The discussion of the learned Pandits is a source of great delight to the audience. They bring neither books, nor notes, nor papers, nor any references when they come for great assemblies where their knowledge in the several branches of Sanskrit will be tested and where they receive due rewards. The greatest Hindu Pandit has hardly anything which deserves the name of a library, whereas the poorest equipped English reader keeps up a decent library. Are not then the heads of these Pandits more valuable than the heads of the greatest English scholars who have to refer to books for constant renewal of their memories ?

Panini’s Ashtadhyayi, Patanjali’s Yogasutram , Jamini’s Jamini Sutras, Pingala’s Chandasastra are some classic works on the sutra type.

The sutra is defined thus:

alpaksharam asandigdham sravat vishvatomukham |
astobham anavadyam ca sutram sutravido viduh ||

Sutra should :

Contain	Alpaksharam	– minimum number of words
Be	Asamdigdham	– Unabiguous
Contain	Sravat	– gist of the topic for which it is meant
Be	Vishvatomukham	– universally valid or be general
Not have	Astobham	– unmeaningful words
	Anavadyam	– be devoid of any fault

Like mathematical formulae, Sutras have a great deal of information in very few words. Sutras are very easy to commit to memory. To verbally communicate the algorithms they are a unique way to put it in sutras (aphorisms).

Evolution of Chandas can be inferred from the following Mundayoka Upanishad (I 1,5)

Rigvedo Yajurvedo Samavedo Atharvanah |
Siksha kalpo vyakarnam niruktam chando jyotisam ||
The study of Vedas (Rig, Yajur, Sama, Atharvana) and Vedangas (Siksha, Kalpa, Vyakaranam, Niruktam, Chandas and Jyotisam) is mentioned. Thus Chandas (Metrics) is a Vedanga.

The date of Maharishi Pingala Chandsutram can be based on date of Vedas. The date of Vedas varies from 8000 BCE to 3500 BCE depending upon astronomical notings, approximate time period of composition of Vedas, as per Max Muller (not on evidences but on his own assumptions). The common assigned date for Pingala Chadasutram is around 200 BCE. This raises a question as to why Vedic seers should take 7800 or 3300 years to formulate Chadasastram. Since the Vedas are already there in metric form, it stands to reason that these rules of chandas should have been first codified and based on these rules, hymns should have been developed. So the age of Maharishi Pingala should be placed not at 200 BCE but much earlier along with age of the Vedas. Such highly refined literature with astronomical data and information could not have been produced by a hoard of barbarians or wandering nomads as has been recorded by present day learned archaeologists and historians. No scholar can ignore a subject just because it is beyond his comprehension. Just by making it that their methodology is scientific, mathematical evidences cannot be ignored. Just because nobody has interpreted the Chandasutram in the lines of binary system, it does not mean that this is not there. Since the sutras are very abstruse and short it takes a lot of effort to decipher and interpret them.

The following algorithms are for the binary system in Pingala Chandasutram.

Chandasutram by Maharishi Pingala contains 18 parichchedas (Sub-chapters) in 8 Adhyayas (main chapters). The 1st parichcheda of six slokas are not sutras. The rest of Chandasutram is composed of sutras.

The fourth sloka is

Maa Ya Raa Sa Taa Ja Bhaa Na La Ga Sammitam Bhramati vangamayam Jagatiyasys |
Sajayathi Pingala Nagah Siva prasadat visudha matih ||

And the sixth sloka

Tri viramam das varnam shanmatramuacha pingala sutram |
Chandovarga padarta pratyaya Hetoschasastaradou ||

In this Maharishi Pingala states that "*Maa, Ya, Raa, Sa, Taa, Ja, Bhaa, Na, La, Ga"* mentioned in the fourth sloka is in itself a sutra, containing ten varnas and specifies that the same is kept on the top of all sutras because it is the basis for "chando varga padartas and pratyayas". Three technical terms are given here Viramam, Matra, Pratyaya. The term pratyaya indicates vast and remarkable meaning. The astonishing wonderful intelligence of Maharishi Pingala is imbibed in various Pratyayas. In fact, the Pratyayas is a collection of extraordinarily ingenious and clever solutions to problems.

The 2nd paricheda gives the pratyahara sutras (general rules). The application of these sutras are universal throughout Chandasutram. If an exception is there, it will be given with it's range of applicability.

1) Dhee Sree Sthree 'm'
2) Varasa 'y'
3) Kaguha 'r'
4) Vasu dha 's'
5) Satekwa 't'
6) ka da sa 'j'
7) kim va da 'bh'
8) na ha sa 'n'
9) Gri 'l'
10) Gante
11) Dhradi parah
12) He
13) Lou sah
14) Glou
15) AstaVasva Iti

Gana is defined as a group of three letters.
Properties of *Maa, Ya, Raa, Sa, Taa, Ja, Bhaa, Na, La, Ga* are explained in the the above sutras.

Dhee Sree Sthree 'm' - meaning three Gurus (long alphabets) make Maa gana and 'm' is Sangana (sign) of it. The word *'Iti'* in the last sutra indicates the ending of first chapter.

Maharishi Pingala has given a very ingenious algorithm to find the index of a sequence of guru-laghu pattern. He has used the three syllable sequence given above which can be treated as octal representation by converting a binary sequence.

Gana	Pingala	Boolean	Index
Maa	G G G	0 0 0	1
Ya	L G G	1 0 0	2
Raa	G L G	0 1 0	3
Sa	L L G	1 1 0	4
Taa	G G L	0 0 1	5
Ja	L G L	1 0 1	6
Bhaa	G L L	0 1 1	7
Na	L L L	1 1 1	8
La			
Ga			

Thus to find the index of the sequence GGGLLG, split it in 3 letter sequence as

GGG 0 0 0 1 maa

LLG 1 1 0 4 sa

And this is read using 'ankanam vamato gatih' - in reverse gives 41 as the index of GGGLLG with the code word ma sa or 1 4. Using this ma and sa coin a word name of the chandas. The reverse process of this gives the sequence for 4 1. The total sequence need not be in multiples of 3. The single or two aksharas at end of grouping is completed using the la (1) and ga (0). This method is the same as the current katapayadi sankya (number) assigned to the melakarta ragas of karnatic music, but in another decimal form. Note that, this is the mirror image of the current Boolean binary representation. A wonderful error-correcting code is given in this. Error-correcting code is a sequence that contains encoded information designed to flag errors in transmission. Each important aspect of a chandas is encoded in the Vedic hymn. Here the rhythm of the hymn and the name of the chandas provide a check on each other.

The 8th Adhyaya gives the following 16 sutras which relate to the Pingala Pratyaya system

1. Prastarah - Algorithms to produce all possible combinations of n binary digits,
   - 8.20 dvikau glau
   - 8.21 misrau ca
   - 8.22 pruthagla mishrah
   - 8.23 vasavastrik
2. Nashtam - Algorithms to recover the missing row,
   - 8.24 l-arddhe
   - 8.25 sa-eke-ga

3. Uddishtam - Algorithms to get the row index of a given row,

8.26 pratilomagunam dvih-l-adyam

8.27 tatah-gi-ekam jahyat

4. Samkhya - Algorithms to get the total number of n bit combinations,

8.28 dvih arddhe

8.29 rupe shunyam

8.30 dvih shunye

8.31 tavadardhe tadgunitam

5. Adhvayoga - Algorithms to compute the total combinations of chandas ranging from 1 syllable to n syllables

8.32 dvih dvihunam tad antanam

8.33 ekone dvah

6. Eka-dvi-adi-l-g-kriya - Algorithms to compute no. of combinations using n – number of syllables taking r - the number of laghus (or gurus), at a time nCr.

8.34 pare purnam

8.35 pare purnam iti

The above algorithms are explained below

1. Algorithm : Prastarah means spreading or expanding. Pingala gives the matrix which shows all the possible combinations of n laghu-guru. There are 2^n possible combinations of n digit binary number. This will result in 2^n * n matrix of all the possible combinations. Pingala gives a recursive algorithm.

8.20 dvikau glau

8.21 misrau ca

8.22 pruthagla mishrah

8.23 vasavastrik

8.20 dvikau glau

With one akshara (binary digit) of guru (G) or laghu (L) we have a prasthara of 2^1 * 1

On putting G as 0 and L as 1 to get similar to the Boolean notation.

G	0
L	1

8.21 misrau ca

The prastara of 2 akshara (binary digits) is got by mixing 1 akshara prastara with itself.

We mix G-L with G and then mix G-L with L. The result is a prasthara of $2^2 * 2$

	Pingala	Boolean
G G	0 0	0 0
L G	1 0	0 1
G L	0 1	1 0
L L	1 1	1 1

The current Boolean notation is the mirror image of this matrix with all 2 digit numbers in ascending order. In Pingala notation the high bits are written to the right of the previous bit. This is same as writing decimal numbers in ancient days. The decimal numbers are written with highest place to the right of a previously written digit. But the reading of the numbers is from right to left as per the dictum 'Ankanam vamato gatih'

8.22 pruthagla mishrah

The procedure for the expansion of 3 akshara (binary digits), mix the prastara of 2 aksharas with G once and L once. The result is a $2^3 * 3$ prastara.

	Pingala	Boolean
G G G	0 0 0	0 0 0
L G G	1 0 0	0 0 1
G L G	0 1 0	0 1 0
L L G	1 1 0	0 1 1
G G L	0 0 1	1 0 0
L G L	1 0 1	1 0 1
G L L	0 1 1	1 1 0
L L L	1 1 1	1 1 1

Thus these sutras give the size of the prastara of triak or 3 bits based the on the previous 3 sutras.

8.23 vasavastriak

This process is to be repeated for higher order prastara. To get all the combinations of 4 akshara, we should know the combinations of 3 akshara, for which 2 akshara combination is necessary and this is got from 1 akshara combination.

	Pingala	Boolean
G G G G	0 0 0 0	0 0 0 0
L G G G	1 0 0 0	0 0 0 1

G L G G	0 1 0 0	0 0 1 0
L L G G	1 1 0 0	0 0 1 1
G G L G	0 0 1 0	0 1 0 0
L G L G	1 0 1 0	0 1 0 1
G L L G	0 1 1 0	0 1 1 0
L L L G	1 1 1 0	0 1 1 1
G G G L	0 0 0 1	1 0 0 0
L G G L	1 0 0 1	1 0 0 1
G L G L	0 1 0 1	1 0 1 0
L L G L	1 1 0 1	1 0 1 1
G G L L	0 0 1 1	1 1 0 0
L G L L	1 0 1 1	1 1 0 1
G L L L	0 1 1 1	1 1 1 0
L L L L	1 1 1 1	1 1 1 1

Thus Maharishi Pingala's algorithm follows naturally from the observation that the list of (n-1) syllable patterns is twice nested within the list of n-syllable patterns. The computer scientist Donald E.Knuth credits this as 'the first ever explicit algorithm for combinatorial sequence generation' meaning Maharishi Pingala was the first to develop a process for systematically listing patters with a given property. Maharishi Pingala's algorithm is recursive – it generates the entire list of n-syllable patterns from the list of (n – 1) syllable patterns. So a list of patterns of any given length can be generated using the same routine. This includes a final stage 1 syllable which is enumerated.
[The Art of Computer Programming, Vol. 4, Fasc 4. Addison Wesley, Upper Saddle River, N.J, 2006. Generating all trees – history of combinatorial generation]

2. Algorithm : Nashtam

8.24 l-arddhe

8.25 sa-eke-ga

These two sutras are to recover the nasht (lost or corrupted row).
Given a row index this is used to get guru-laghu combination or the binary combination.
Write L (Laghu), if the given index number is halved without a remainder,
Otherwise write G (Guru), add 1 to the index number and then halve it.
The recursive algorithm is, given an index I, remove the first syllable of the pattern, to get pattern for index I -1

Index(I – 1) = Index(I /2) if I is even or Index(I +1)/2 if I is odd

Eg: To get the guru-laghu combination for the row 6 of 3 akshara prastara.
The row index number 6, when halved gives no remainder. So put L or 1
Next we have 3, when halved gives a remainder. So add 1 and halve. Put G or 0.
Next we have 2, when halved gives no remainder. So put L or 1

The 6^{th} row is L G L   1 0 1

The mirror image of 101 is also 101 whose value is 5.

If the given row index is odd which gives a remainder we put G or 0, add 1 and then halve it.

Thus we can see that the row index is 1 more than decimal value given by the binary digit pattern representation. So Maharishi Pingala's binary values start from 0. But the index starts with 1. These two are different. This algorithm can also be used to convert a decimal number to binary.

3. Algorithm: Uddishtam

From a given prastara to get the uddishtam (desired) row index without counting from the top row. Given a G-L combination to get the row index. This is the inverse method of nashtam

8.26 pratilomagunam dvih-l-adyam

8.27 tatah-gi-ekam jahyat

pratilomagunam dvih-l-adyam – In the reverse starting from the l-adyam (first laghu) multiply by dvih (2).

tatah-gi – if the akshara is Guru,

ekam jahyat – subtract 1 after multiplying by 2.

The 8.27 sutra is to be reconstructed as anuvritti. The word dvih is not there in this, but has to be taken from the sutra 8.26. The start is always 1 and we start from a L.

Find the row index of G G L

Start with 1 and the right most L .

1 x 2 = 2 this is L so multiply by 2

2 x 2 = 4 this is G So subtract 1 = 3

3 x 2 = 6 this is G So subtract 1 = 5

To get the decimal value of a number in any base, these set of rules can be used.

4. Algorithm : Sankhya

'Sankhya' stands for the number of possible combinations of n bits. Here Pingala gives a method which appears to be strange and complicated, but is based on a very ingenious method of simplification of the purva or power or exponent. This results in the reduction of number of operations needed for calculating sankhya.

For example

$$2^{11} = 2^{10} \times 2 = (2^5)^2 \times 2 = (2^4 \times 2)^2 \times 2 = [(2^2)^2 \times 2]^2 \times 2$$

The form requires just 3 multiplication by the same number ie squaring and 2 multiplications by the number, here in this case 2. In Vedic mathematics Ganita Sutras the method of squaring any number

of digit numbers is given as straight squaring. And this simplifies to find the binary of the exponent and using it to know when to raise to power 2 (square) or multiply by 2.

8.28 dvih arddhe
8.29 rupe shunyam
8.30 dvih shunye

8.31 tavadardhe tadgunitam
If the number is divisible by 2 (arddhe) divide by 2 and write 2 (dvih)
Else subtract 1 (rupe) and write 0 (shunyam).
If the answer were 0 (shunya), multiply by 2 (dvih) and if the answer
Were 2 (arddhe) multiply (tad gunitam) by itself (tavad).

Example

16	2	(Even, divide by 2 and write 2)
8	2	(Even, divide by 2 and write 2)
4	2	(Even, divide by 2 and write 2)
2	2	(Even, divide by 2 and write 2)
1		
0	0	(Odd, subtract 1 and write 0)

Start in the 2nd column, from bottom to top.

0	1 x 2 = 2	(if 0, multiply 1 by 2)
2	2 x 2 = 4	(if 2, multiply by itself ie square it)
2	4 x 4 = 16	
2	16 x 16 = 256	
2	256 x 256 = 65536	

This algorithm for calculating nth power of 2 is a recursive algorithm and its complexity is O(log2n), whereas the complexity of calculating power by normal multiplication is O(n).

5. Algorithm : Adhvayoga

8.32 dvih dvihunam tad antanam
8.33 ekone dvah

This sutra gives the adhvayoga of all the chandas (adhva) with
number of syllables less than or equal to n.
To get adhvayoga, multiply the last one (tat antanam) by 2 (dvih) and then
subtract once 2 (ekone dvah).

That is

Yoga (sum(2^n))= 2^n x 2 - 2 = 2^{n+1} - 2

6. Algorithm : Eka-dvi-adi l-g kriya

Procedure to calculate nCr.

Pingala's sutra are very cryptic.

8.34 pare purnam

8.35 pare purnam iti

The sutra 8.34 means "complete it using the two far ends pare'

Start with 1 in a cell. Under this cell draw two cells. Then fill all the cells which are at the far ends, in each row by

1. Meru is constructed like this:

								1								
							1		1							
						1		2		1						
					1		3		3		1					
				1		4				4		1				
			1		5						5		1			
		1		6								6		1		
	1		7										7		1	
1		8												8		1

The next sutra 8.35 says to complete a cell using above 2 cells, filling again the end cells.

								1								
							1		1							
						1		2		1						
					1		3		3		1					
				1		4		6		4		1				
			1		5		10		10		5		1			
		1		6		15				15		6		1		
	1		7		21						21		7		1	
1		8		28								28		8		1

Repeat the above procedure to get

								1								
							1		1							
						1		2		1						
					1		3		3		1					
				1		4		6		4		1				
			1		5		10		10		5		1			
		1		6		15		20		15		6		1		
	1		7		21		35		35		21		7		1	
1		8		28		56		70		56		28		8		1

Highlights of Maharishi Pingala's algorithms:

Maharishi Pingala has used recursion extensively to describe the algorithms. Further, he uses stack to store the information of intermediate operations. All these algorithms use a terminating condition also, ensuring that the recursion terminates. Recursive algorithms are easy to conceptualize, and implement mechanically.

The sutra style was prevalent in Bharat, and unlike modern mathematics, the Bharatheeya mathematics was passed through generations orally. Sutras being very brief, and compact, were easy to memorize and also communicate orally. In which hymns of Vedas these encoding is done is not available. So a serious search of the Vedas as binary encoding is being done by me, using the computer to deciper the binary numbers and their linkage to the various astronomical events recorded in the Vedas. The results are very astounding. Whole astronomical tables are binary encoded and given. The Vedanga Jyotisham of the Vedas contain the binary encoded astronomical numbers. The Vedic mathematics based on Ganita sutras, as given to us by H.H.Jagadguru Sankaracharya Bharti Tirtaji Maharaj is similar to modern discrete mathematics clearly outlining a metatheory of calculation, systems of decimal, binary, ternary, sexadecimal calculation, an advanced combinatorics, algebra of logic for description of any system based on a binary digits also. Most researchers, who recognize the important contribution of Hindus, their more advanced concept of zero and hundreds of practical mathematical algorithms, mostly underestimate the value of the ancient Vedic mathematics owing to their ignorance of the Vedic Sanskrit sources. The purpose of Chandasutram is not only to enumerate of millions of syllabic chandas and to determine the serial order, but to represent an algebraic binary code developed by means of Vedic Sanskrit, a highly perfected artificial intelligence programming language for encoding large decimal numbers which occur in astronomical calculations. Advantages of an algebraic binary-octal (tristub) system offered by Maharishi Pingala are indisputable in the field of astronomical calculus requiring great accuracy and handling of extra-large numbers. Chandasutram is the science of a perfect metacode organized on the basis of the theory of sets, arithmetic and geometric progressions, combinatorial analysis, theory of infinitesimal and indefinitely large numbers, which allows to combine numbers of different levels of meaning, exact and different systems of calculation and astronomical information encoded in digital form. This ancient classification of the meters based on a developed combinatorial analysis and theory of sets is one of the outstanding achievements of the ancient Vedic seers and should impress the mathematician, astronomer and the specialist in metrics. The fact that Maharishi Pingala used a precise and detailed classification of the binary codes, eight thousand years ago, is highly mysterious and intriguing.

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Aryabhata: The Great Indian Mathematician and Astronomer

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Introduction

Astronomy is one of the oldest sciences. India has the tradition of excelling in the study of celestial objects from the ancient past to till date. The first golden age for Astronomy started after the invention of telescope in the early part of seventeenth Century CE. Evidences for the heliocentric solar system model, discovery of sunspots, discovery of satellites of planets and many more were found after that. However, in 5th century CE, by simply observing the heavens with unaided eyes and by using indigenous mathematical tools a 23 year youngster in India accurately measured the period of Earth's rotation with respect to the fixed stars in the sky, deduced the shape of the Earth and its rotation, and made many remarkable achievements in Astronomy and Mathematics. That was Aryabhata (476–550 CE) the great Indian Mathematician.

Aryabhata carried out intense researches on mathematics and astronomy. Findings of his major works were presented in his book 'Aryabhatiya'.

In the field of Mathematics, he contributed in the development of algebra, plane trigonometry, and spherical trigonometry. Aryabhata made no specific mention of the place of his birth. But Bhaskara I, his follower and commentator indicates his place of birth southern Gujarat or Northern Maharashtra (Asmaka Country). There is a sufggestion to the effect that Kerala could have been the birth place of Aryabhatta. He was believed to have flourished at Pataliputra (Patna) in the ancient Magadha Country.

Mathematics

The place-value system, first seen in the 3rd century Bakhshali Manuscript, was clearly in place in his work. Though he did not use a symbol for zero, the knowledge of zero was implicit in Aryabhata's place-value system as a place holder for the powers of ten with null coefficients However, Aryabhata did not use the brahmi numerals. Continuing the Sanskritic tradition from Vedic times, he used letters of the alphabet to denote numbers, expressing quantities, such as the table of sines in a mnemonic form.

π is a mathematical constant whose value is the ratio of any circle's circumference to its diameter in Euclidean space. This is the same value as the ratio of a circle's area to the square of its radius. It is approximately equal to 3.141593 in the usual decimal notation. Many formulae from mathematics, science, and engineering involve π, which is one of the most important mathematical and physical constants. Unlike many physical constants, pi is a dimensionless quantity which means that it is simply a number without physical units.

π is an irrational number, which means that its value cannot be expressed exactly as a fraction m/n, where m and n are integers. Consequently, its decimal representation never ends or repeats. It is also a transcendental number, which implies, among other things, that no finite sequence of algebraic operations on integers (powers, roots, sums, etc.) can be equal to its value; proving this was a late achievement in mathematical history and a significant result of 19th century German mathematics. Throughout the history of mathematics, there has been much effort to determine π more accurately and to understand its nature; fascination with the number has even carried over into non-mathematical culture.

Aryabhata gave an accurate approximation for π. He wrote in the Aryabhatiya the following:-
Add four to one hundred, multiply by eight and then add sixty-two thousand, the result is approximately the circumference of a circle of diameter twenty thousand. By this rule the relation of the circumference to diameter is given.

This gives π = 62832/20000 = 3.1416 which is a surprisingly accurate value. In fact π = 3.14159265 correct to 8 places. If obtaining a value this accurate is surprising. Another interesting paper discussing this accurate value of ð by Aryabhata is by P. Jha, (Aryabhata I and the value of π, *Math. Ed. (Siwan)* **16** (3) (1982), 54-59) where Jha writes:-

"Aryabhata I's value of π is a very close approximation to the modern value and the most accurate among those of the ancients. There are reasons to believe that Aryabhata devised a particular method for finding this value. It is shown with sufficient grounds that Aryabhata himself used it, and several later Indian mathematicians and even the Arabs adopted it. The conjecture that Aryabhata's value of π is of Greek origin is critically examined and is found to be without foundation. Aryabhata discovered this value independently and also realised that π is an irrational number. He had the Indian background, no doubt, but excelled all his predecessors in evaluating π. Thus the credit of discovering this exact value of π may be ascribed to the celebrated mathematician, Aryabhata I. " Aryabhata may have come to the conclusion that π is irrational. The irrationality of pi was proved in Europe only in 1761. In the second part of the Aryabhatiyam (gaṇitapâda 10), he writes:

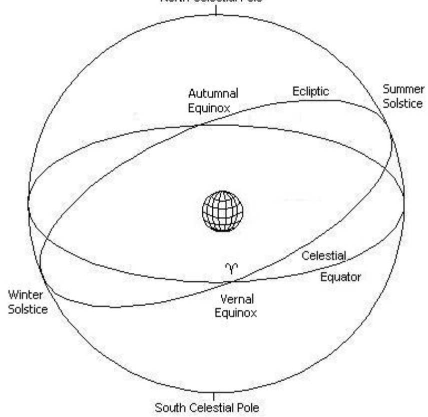
caturadhikam śatamaṣṭaguṇam dvāṣaṣṭistathā sahasrāṇām
ayutadvayaviṣkambhasyāsannō vṛttapariṇāhāḥ

"Add four to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of 20,000 can be approached." This implies that the ratio of the circumference to the diameter is ((4+100)×8+62000)/20000 = 62832/20000 = 3.1416, which is accurate to five significant figures.

Astronomy

The great Indian mathematician Aryabhata was a pioneer of mathematical astronomy. He describes the earth as being spherical and that it rotates on its axis, among other things in his work Aryabhatiya. Aryabhatiya is divided into four sections. Gitika, Ganitha (mathematics), Kalakriya (reckoning of time) and Gola (celestial sphere). The discovery that the earth rotates on its own axis from west to east is described in Aryabhatiya (Gitika 3,6; Kalakriya 5; Gola 9,10;). For example he explained the apparent motion of heavenly bodies is only an illusion (Gola 9), with the following metaphor. "Just as a passenger in a boat moving downstream sees the stationary (trees on the river banks) as traversing upstream, so does an observer on earth see the fixed stars as moving towards the west at exactly the same speed (at which the earth moves from west to east.) "

Equatorial System



The position of any point on the surface of the sphere (and hence that of any celestial body which is referred to it) can be given with reference to the equator or the ecliptic. In the equatorial co-ordinate system, position is specified by right ascension and declination. Right ascension (α) is the angular distance along the equator from the vernal equinox; declination (δ) is the distance north or south of the equator along a great circle passing through the point in question and the two celestial poles. In the ecliptic co-ordinate system, position is specified by celestial longitude and latitude. Celestial longitude is the angular distance along the ecliptic, again from the vernal equinox; celestial latitude is the distance north or south of the ecliptic along a great circle passing through the two celestial poles.

Celestial objects are at constant RA, but change their hour angle as time proceeds. If measured in units of hours, minutes and seconds, HA will change for the same amount as the elapsed time interval is, as measured in *star time* (sidereal time ST), which is defined so that a sidereal rotation of Earth takes 24 hours star time, which corresponds to 23 h 56 m 4.091 s standard (mean solar) time. This is actually the reason why RA and HA are measured in time units. The standard convention is that HA is measured from east to west so that it increases with time, and this is opposite to the convention for RA

Star time is ST = 0 h by definition whenever the vernal equinox, RA = 0 h, crosses the local meridian, HA = 0. As time proceeds, RA stays constant, and both HA and ST grow by the amount of time elapsed, thus star time is always equal to the hour angle of the vernal equinox. Moreover, objects with "later" RA come into the meridian HA = 0, more precisely with RA which is later by the amount of elapsed star time, so that also star time is equal to the current Right Ascension of the local meridian.

Sidereal Time

As the Earth moves in its orbit around the Sun, the Earth must rotate more than 360 degrees in one **solar day** . A **solar day** lasts from when the Sun is on the meridian at a point on Earth until it is next on the meridian. A solar day is exactly 24 hours (of solar time). Because of the Earth's revolution, a solar day is slightly longer than a sidereal day. In every day life, we use solar time. The Earth must rotate an extra 0.986 degrees between solar crossings of the meridian. Therefore in 24 hours of solar time, the Earth rotates 360.986 degrees. Because the stars are so distant from us, the motion of the Earth in its orbit makes an negligible difference in the direction to the stars. Hence, the Earth rotates 360 degrees in one **sidereal day** .

A **sidereal day** lasts from when a distant star is on the meridian at a point on Earth until it is next on the meridian. A sidereal day lasts 23 hours and 56 minutes (of solar time), about 4 minutes less than a solar day

In timing celestial events, it is convenient to use the rotation rate of the earth as the basis of the length of the day. The vernal equinox or the first point of Aries is the reference point chosen on the rotating celestial sphere to define a sidereal day. A **sidereal day** is defined as the time interval between two consecutive passages of the vernal equinox over the observer's meridian.

Aryabhata calculated the sidereal rotation (the rotation of the earth referencing the fixed stars) as 23 hours, 56 minutes, and 4.1 seconds; the modern value is 23:56:4.091. Similarly, his value for the length of the sidereal year at 365 days, 6 hours, 12 minutes, and 30 seconds is an error of 3 minutes and 20 seconds over the length of a year. The notion of sidereal time was known in most other astronomical systems of the time, but this computation was likely the most accurate of the period

In Aryabhata's system of astronomy days are reckoned from uday, dawn at lanka or "equator". In some texts, he ascribes the apparent motions of the heavens to the Earth's rotation.

Kepler (1571 – 1630 CE), a German astronomer, was investigating the motion of planets at about the same time as Galileo was looking at them through the telescope. He was convinced that the heliocentric

model was the correct one to describe planetary motion. He was analysing the accurate observations of planet positions compiled by Tycho Brahe. Although Kepler started with circular orbits he just could not fit the observed data for Mars to a circular orbit. He was forced to assume an elliptical orbit. Further work with the accurate observational data led to the famous three laws of Kepler. First Law: The orbit of each planet is an ellipse, with the sun at one focus. Thus the distance of the planet from the sun varies as it moves in its orbit. However, over thousand years prior to Kepler, Aryabhata discovered the nature of the orbits of the planets. He gives a systematic treatment of the position of the planets in space. Aryabhata gives the radius of planets in terms of the Earth-Sun distance as essentially their periods of rotation around the Sun. Astonishingly he mentions that the orbits of the planets are ellipses.

As mentioned earlier, Aryabhata claimed that the Earth turns on its own axis, and some elements of his planetary epicyclic models rotate at the same speed as the motion of the Earth around the Sun. The planetary orbits were also given with respect to the Sun and he also states: "Whoever knows this Dasagitika Sutra which describes the movements of the Earth and the planets in the sphere of the asterisms passes through the paths of the planets and asterisms and goes to the higher Brahman." Thus, it has been suggested that Aryabhata's calculations were based on an underlying heliocentric model, in which the planets orbit the Sun.

As the Earth is not perfectly spherical, no single value serves as its natural diameter. Instead, being nearly spherical, a range of values from 39922 km to 40054 km spans all proposed diameters according to need, and several different ways of modeling the Earth as a sphere all yield a convenient mean radius of 40010 km ($\approx$ 3,959 mi).

Aryabhatiya also estimates the circumference of Earth, accurate to 1% which is remarkable. He gave the circumference of the earth as 4 967 yojanas and its diameter as 1 5811/24 yojanas. Since 1 yojana = 8 k.m. this gives the circumference as 39966 km, which is an excellent approximation to the currently accepted value of 40074 kms.

He also gave the correct explanation of lunar and solar eclipses and that the Moon shines by reflecting sunlight. Aryabhata states that the Moon and planets shine by reflected sunlight. Instead of the prevailing cosmogony in which eclipses were caused by pseudo-planetary nodes Rahu and Ketu, he explains eclipses in terms of shadows cast by and falling on Earth. Thus, the lunar eclipse occurs when the moon enters into the Earth's shadow (verse gola.37). He discusses at length the size and extent of the Earth's shadow (verses gola.38–48) and then provides the computation and the size of the eclipsed part during an eclipse. Later Indian astronomers improved on the calculations, but Aryabhata's methods provided the core. His computational paradigm was so accurate that 18th century scientist Guillaume Le Gentil, during a visit to Pondicherry, India, found the Indian computations of the duration of the lunar eclipse of 30 August 1765 to be short by 41 seconds, whereas his charts (by Tobias Mayer, 1752) were long by 68 seconds.

Tropical and Sidereal year

The year used in civil life is the **tropical year** and is defined as **the interval in time between consecutive passage of the sun through the vernal equinox**. It is equal to 365.24220 mean solar days. For convenience there are an integral number of days in a year. 365 days for three years followed by a leap year of 366 days. Any year whose number is divisible by 4 is a leap year. The exception is the century years. A century year is a leap year only if the number of hundreds in the century is divisible by 4. Thus the year 1900 was not a leap year but the year 2000 was and 2100 will not be a leap year. The Julian Calendar, which preceded the present one, did not have this last rule and gave the length of the tropical year as 365.25 mean solar days. By 1582 the difference between this number and the actual length of the tropical year had piled up to about 12 days. It is then that the present Gregorian calendar was introduced removing this error.

The **sidereal year is defined as the time interval between successive passage of the mean sun in front of the same star background.** This is equal to 365.25636days (~365 days 6 hours). Thus the sidereal year is about 20.4 minutes longer than the tropical year.

It was noticed that the tropical year was shorter than the sidereal year by nearly 20.4 minutes. Notice that the tropical tear is defined as the successive passage of the mean sun through the vernal equinox, while the sidereal year brings the sun back to the same background of stars. **The difference between the two definitions of years implies that the vernal equinox γ is itself shifting from east to west with respect to the stars**. This shift is 50.2 seconds of arc in one year. After the continuous works of astronomers and mathematicians and repeated observations using sophisticated telescopes, the above value was deduced. However, as early as 5th Century CE, based on the visual observations, Aryabata deduced the length of the year. His value for the length of the year at 365 days 6 hours 12 minutes 30 seconds is amazingly very close to the true value 365 days 6 hours.

Conclusion

After the advent of modern equipments to observe the heavens, computational facilities and space age, humankind is now bestowed with better knowledge about the Universe we live-in and also we got sharper tools for computations. However, during the late 5th Century and the early party of 6th Century CE the lone Giant Indian Astronomer and Mathematician Aryabhata, without any tools made significant contribution to the studies of Mathematics and Astronomy. Bhaskara I who wrote a commentary on the Aryabhatiya about 100 years later wrote of Aryabhata:-

Aryabhata is the master who, after reaching the furthest shores and plumbing the inmost depths of the sea of ultimate knowledge of mathematics, kinematics and spherics, handed over the three sciences to the learned world.

Newton said that he was standing on the shoulders of the giants. The young learners must study the contributions of the Giant 'Aryabhata' to attain knowledge. The tradition of excelling in the field of Astronomy and Mathematics by Indian Scientists is continuing till date. Without learning the contributions by the Indian Astronomers like S. Chandrasekhar, Megnad Saha one can not become a true astronomer. This tradition must continue. The primary and higher level curriculum must contain suitable portions to boost Indian students ability in these subjects.

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Varahamihira : A Versatile Genius

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[Abbreviations used
B.S.: Bṛhatsaṁhitā, B.J.: Bṛhajjātaka, S.S.: Siddhāntas'iromaṇi, P.S.: Pañcasiddhāntikā]

1. Introduction

Baṭṭotpala invokes the Sun at the commencement of his commentary on Bṛhajjātaka [3] thus:

yac chāstram savitā cakāra vipulaṁ skandhaistribhir jyotiṣaṁ
tasyocchittibhayāt punaḥ kaliyuge samsṛtya yo bhūtalam |
bhuyaḥ svalpataraṁ Varāhamihira vyājena sarvaṁ vyadhād
ittham yam pravadanti mokṣa kus'alās tasmai namo bhāsvate ||

The three-fold *Jyotiṣa* was propounded by the Sun. Fearing that it would be lost in Kaliyuga, the Sun took the incarnation in the world as Varāhamihira and expounded the three branches in a short form. This is what people with knowledge of salvation say. Obeisance to that Sun.

The statement is fully justified by the fact that Varāhamihira wrote on all the three branches of *Jyotiṣa* viz *Siddhānta*, *Horā* and *Saṁhitā*. In *Siddhānta* he wrote *Pañcasiddhāntikā* dealing with the five systems of astronomy namely *Vāsiṣṭha*, *Paitāmaha*, *Saura*, *Romaka* and *Paulis'a*. Bhaṭṭotpala says that he wrote an abridged version of this also.

It gives in eighteen chapters a detailed exposition of the systems. In *Horā*, he wrote *Bṛhajjātaka* and *Laghujātaka*. These deal with horoscopy. *Bṛhatsaṁhitā* is really encyclopaedic in its range and deals with a wide variety of topics covering the whole gamut of terrestrial and celestial phenomena. The topics include transits of planets, comets, architecture, iconography, omens, cosmetics, water-driving, aphrodisiacs, horticulture, species of men and women, weather forecast, details of Indian geography etc. He also wrote an abridged version called *Samāsasaṁhitā* known only through quotations. *Vaṭakaṇikā* is a work on omens, exclusively. But it exists only in a fragmentary form as quoted in other works. On travels or what may be called military astrology he wrote *Bṛhadyātrā*, *Svalpayātrā* and *Yogayātrā*. He wrote *Bṛhadvivāhapaṭala* on marital horoscopy. Bhaṭṭotpala mentions that he also wrote *Svalpavivāhapaṭala*. More than thirty works are attributed to him, but the authenticity cannot be asserted. Apart from the global sweep of learning he had, he was a poet who conveyed meanings through *metres*[1] and even indulged in a metrical extravaganza in *Bṛhatsaṁhitā* while describing the transits[B.S.104]. In fact, no other astronomer of India possessed his versatility.

He belonged to the latter part of the fifth century A.D. and the major part of 6th century A.D. Āmarāja mentions in his commentary on *Bṛahmasphuṭasiddhānta* that Varāhamihira died in S'aka 509 (587 A.D.). Not much is known about him. He mentions in *Bṛhajjātaka* [2] thus:

ādityadāsa tanayastadavāptabodhaḥ
kāpiṣṭhilaḥ savitṛlabdhavaraprasādaḥ |
āvanitiko munimatānyavalokya samyag
ghorāṁ varāhamihiro rucirāṁ cakāra ||

B.J. 26.1

This implies that Varāhamihira was the son of Ādityadāsa, and belonged to Kapiṣṭhala gotra. He got knowledge from him and received the blessings of the Sun. He was an inhabitant of Avantī. Some texts read Kāpitthaka instead of Kāpiṣṭhilah and using this, some scholars interpret that he belonged to Kapitthaka village. He was a worshipper of the Sun. He has invoked the Sun in most of his works save possibly *Vivāhapaṭala* in which he worships Kāmadeva and *Laghujātaka* in which he worships S'iva. Though he invokes the Sun in *Bṛhatsaṁhitā* at the commencement he has described Viṣnu as the supreme being:[B.S.43.1-4]

labdhavarāḥ kṣīrodaṁ gaṭvā te tuṣṭuvuḥ surāḥ sendrāḥ |
srivatsāṅkaṁ kaustubhamaṇi kiraṇodbhā sitoraskam ||
s'rīpatimacintyamasamaṁ samantataḥ sarvadehinaṁ sūkṣmam|
paramātmanamanādiṁ viṣṇumavijñāta paryantam ||

After getting the boon from Brahma, Indra with other devas, reached the milky ocean and extolled Viṣṇu, the consort of Lakṣmi, adorned with Srivatsa and breast shining with the rays of Kaustubha gem, one who is incomprehensible, unequalled, the inner-self of all beings, the Supreme Atman and one whose beginning and end are not known. Though he worshipped the Sun following the tradition of the family, he was really an enlightened philosopher.

During the time of Varāhamihira the science of *Jyotiṣa* propounded by the eighteen sages was in shambles. He revived the learning by writing monumental treatises on the three branches of *Jyotiṣa* and stimulated interest in the later scientists. No wonder, Bhāskara II [S.S.1.2] makes a reference to him with reverence. From the verse:

dhanvantarikṣapaṇakāmarasiṁhas'aṅku
vetālabhaṭṭaghaṭakarparakālidāsāḥ |
khyāto varāhamihiro nṛpateḥ sabhayāṁ
ratnāni vai vararuci rnavikramasya ||

describing the nine gems in the Court of king Vikramāditya, one may conclude that he was in the Court of Candragupta II. But this is sheer anachronism as the periods do not tally. From his reference to Pṛthuyas'āḥ[B.J. 11.5] Pṛthuguṇayas'āḥ[B.J. 11.10] and naming his son Pṛtuyas'as it appears that he was a contemporary of Vainya Gupta, Pṛthu and Vainya being synonymous.

2. Contributions of Varāhamihira

(i) *Pañcasiddhāntikā*

We shall first discuss his main astronomical work *Pañcasiddhāntikā*. As observed early the five systems of astronomy are discussed in this.

Varāhamihira observes,

paulis'a tithiḥ sphuṭoSyaṁ
tatsyāsannastu romakaḥ proktaḥ |
spaṣṭataraḥ sāvitraḥ
paris'eṣau dūravibhraṣṭau ||

P.S. 1.4

This means that tithi given by Paulis'a is correct, Romaka comes close to that Saura is more accurate. Others are to be rejected.

Still Varāhamihira has discussed all these systems, because he thought each system is important and could be rectified by effort.

One important contribution of Varāhamihira is to Trigonometry. In ancient days trigonometrical functions were defined in the following way: (see fig 1).

Consider a circle of radius R, with centre O. Let A′OA and B′OB be two diameters intersecting at right angles, one being horizontal and the other vertical. Let AC be an arc such that $\angle AOC = \theta$. Then the arc AC is measured by θ.

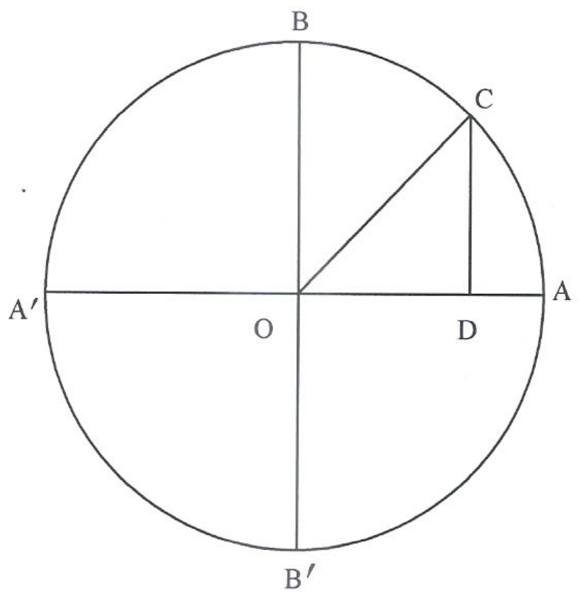


Fig: 1

Draw CD perpendicular to OA. Then CD is called the *bhujajyā* and OD is called *koṭījyā*.

Then $CD = R \sin \theta$ and $OD = R \cos \theta$.

One can easily verify that

$$(bhujajy\bar{a})^2 + (kot\bar{i}jy\bar{a})^2 = R^2$$

bhujajyā θ = *koṭījyā* (90° − θ) and consequently *koṭījyā* θ = *bhujajyā* (90° − θ).

We shall prove the following result found in Pañcasiddhāntikā:

$$R^2 \sin^2 \theta = \frac{R}{2}\left[R - R\sin\left(90^\circ - 2\theta\right)\right] \text{ see[6]}$$

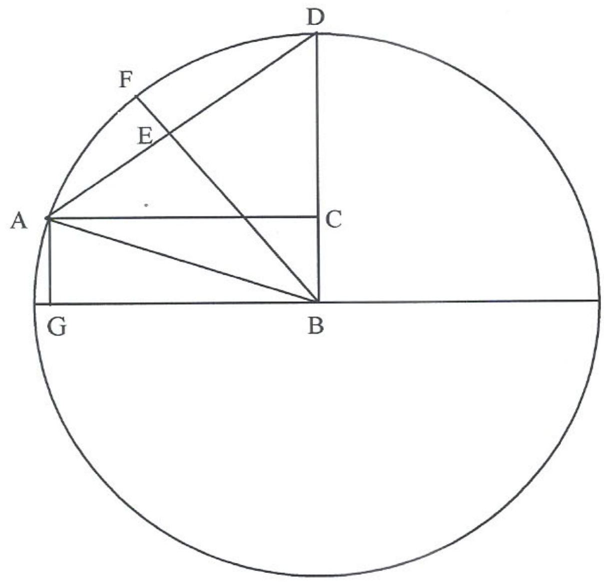


Fig: 2

Let B be the centre of a circle and AB, DB radii and $\angle ABD = 2\theta$. The bisector of $\angle ABD$ meets AD at E and the circle at F. Draw AC perpendicular to DB and AG perpendicular to diameter through B perpendicular to BD.

We have $DE = R \sin \theta$

Therefore,

$$R^2 \sin^2 \theta = DE^2 = \frac{1}{4} DA^2 = \frac{1}{4}\left(AC^2 + CD^2\right)$$

Now,

$AC = R \sin 2\theta$ and therefore

$AC^2 = R^2 \sin^2 2\theta$ and

$$CD^2 = (BD - BC)^2 = (BD - AG)^2$$

$$= \left[R - R\sin\left(90° - 2\theta\right)\right]^2$$

Therefore,

$$R^2 \sin^2 \theta = \frac{1}{4}\left(AC^2 + CD^2\right)$$

$$= \frac{1}{4}\left[R^2 \sin^2 2\theta + \left\{R - R\sin\left(90° - 2\theta\right)\right\}^2\right]$$

$$= \frac{1}{4}\left[R^2 \sin^2 2\theta + R^2 + R^2 \sin^2\left(90° - 2\theta\right) - 2R^2 \sin\left(90° - 2\theta\right)\right]$$

$$= \frac{2 \times R}{4}\left[R - R\sin\left(90° - 2\theta\right)\right]$$

$$= \frac{R}{2}\left[R - R\sin\left(90° - 2\theta\right)\right]$$

Varāhamihira took R = 120′ whereas Āryabhaṭa took it as 3438′.

From these we can infer that Varāhamihira was aware of the following:

1. $\sin\left(90° - \theta\right) = \cos\theta$
2. $\sin^2\theta + \cos^2\theta = 1$
3. $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

The values R sin 30°, R sin 45° and R sin 60° were known geometrically and using the formulae above R sin θ and R cos θ of the angles (arcs) from 0 to 90° with the interval of 3°45′ can be obtained. His table runs thus:

θ	120′ sin θ	sin θ	Modern Values
3°45′	7′51″	0.06542	0.06540
7°30′	15′40″	0.13056	0.13053
11°15′	23′25″	0.19514	0.19509
15°	31′4″	0.25889	0.25882
18°45′	38′34″	0.32139	0.32143
22°30′	45′56″	0.38278	0.38268
26°15′	53′5″	0.44236	0.44229
30°	60′	0.50000	0.50000
33°45′	66′40″	0.55556	0.55556
37°30′	73′3″	0.60875	0.60876
41°15′	79′7″	0.65931	0.65935
45°	84′51″	0.70708	0.70711
48°45′	90′13″	0.75181	0.75184
52°30′	95′13″	0.79347	0.79335
56°15′	99′46″	0.83139	0.83147
60°	103′56″	0.86611	0.86602
63°45′	107′38″	0.89694	0.89687
67°30′	110′53″	0.92402	0.92388
71°15′	113′38″	0.94694	0.94693
75°	115′56″	0.96611	0.96593
78°45′	117′43″	0.98097	0.98079
82°30′	119′	0.99167	0.99144
86°15′	119′45″	0.99792	0.99786
90°	120′	1.0000	1.0000

Āryabhaṭa used 3438′ instead of 120′ and 3438′ is nearly equal to $\frac{180}{\pi}\times 60$. Āryabhaṭa's approximation for π was 3.1416 whereas Varāhamihira took it as $\sqrt{10}$. These illustrate the mathematical content of *Pañcasiddhāntikā*.

Before discussing the astronomical content of *Pañcasiddhāntikā* we note that the systems considered are Vāsiṣṭha, Paitāmaha, Saura, Romaka and Paulis'a.

While commenting on P.S. 1.3 Kuppanna Sastri observes,

> "Of the *siddhāntas* Brahma is the author of Paitāmaha, Vasiṣṭha that of *Vāsiṣṭha*, Paulis'a that of *Paulis'a*, Romaka that of *Romaka* and Sūrya that of *Saura*. From a dialogue between Sūrya and Aruṇa, it can be learnt that the five *siddhāntas* were given to their respective recipients. According to tradition at the first instance Brahma saw this lore of astronomy embedded in the Vedas and extracted in the form of Paitāmaha. He taught this to his son Vasiṣṭha at the behest of Viṣṇu and again to Sūrya who was created with the express purpose of giving Time to the Universe. Vasiṣṭha gave this lore to his son, Parāsara who in turn gave *Parās'arasiddhānta* to sages. One sage Paulis'a taught this to the sages Garga etc. Sūrya himself being born among the *yavanas* by the curse of Brahma taught this science to Romaka and Romaka propounded *Romakasiddhānta*. Thus these five *siddhāntas* are the most ancient."

Even these systems were lost and as given by Varāhamihira these were only in the revised forms. See [6] for details.

Several topics treated are common to different *siddhāntas*.

Ahargaṇa is the number of days that have elapsed since the commencement of the epoch. Varāhamihira gives it for Paulis'a [P.S.1-11-13]. Romaka [P.S.1, 8-10] and Paitāmaha [P.S.12-1-2]. In this way, *Nakṣatra-tithi*, *Ravisphuṭa*, *Candrasphuṭa*, *Aharmāna* (duration of the day) *Vyatīpāta*, *Vaidhṛti*, Solar and Lunar eclipses, and heliacal risings have been discussed in different systems. But Saura alone uses epicyclic theory and the geometrical model. Others do not suggest that.

For the computation of planetary positions, mainly two methods were used, the arithmetical model in which extrapolation is used and the geometrical model in which trigonometry is applied. We shall illustrate this first from *Vāsiṣṭhasiddhānta*. The method of finding the longitude of the Sun is given thus:

> Multiply the days from the epoch by 4 and add 6. Divide this by 1461 and take the remainder. Take from this successively, the quantity 126 reduced by 1, 0, 0, 0, 2, 4, 7, 9, 9, 8, 6, 5 (the twelve quantities 125, 126, 126, 126, 124, 122, 119, 117, 117, 118, 120, 121.). The Sun's rās'i Meṣa etc are successively obtained.
>
> P.S. 2.1

This is fairly a simple method without any sophistication. In the early days this would have been sufficient. In a similar way we can find the Moon's position. This method was improved later. In *Pañcabodha*[6], we get *Yogyādivākyas* and *Cāndravākyas* which improve this system.

In *Saurasiddhānta*, we come across the geometrical model. For this we imagine a mean planet moving in *kakṣyāvṛtta* and introduce *manda* and *s'īghravṛttas* to correct the mean position.

We first discuss the motion of the Sun and the Moon. According to the epicycle theory, the mean Sun (Moon) moves in the *kakṣyāvṛtta* and the true Sun (Moon) moves in a circle (*mandavṛtta*) with the mean Sun (Moon) at the centre completing one revolution in the same time as the mean Sun (Moon) but in the opposite direction. In the figure (fig: 3) A is the Apogee and S_1 the mean Sun when at Apogee, true and mean positions coincide. After some time the true position is S_2 and we need the correction $\angle S_1AS_2$ to get the true position. The circumference of the *kakṣyāvṛtta* is 360° and that of *mandavṛitta* (*mandaparidhi*) is given in degrees.

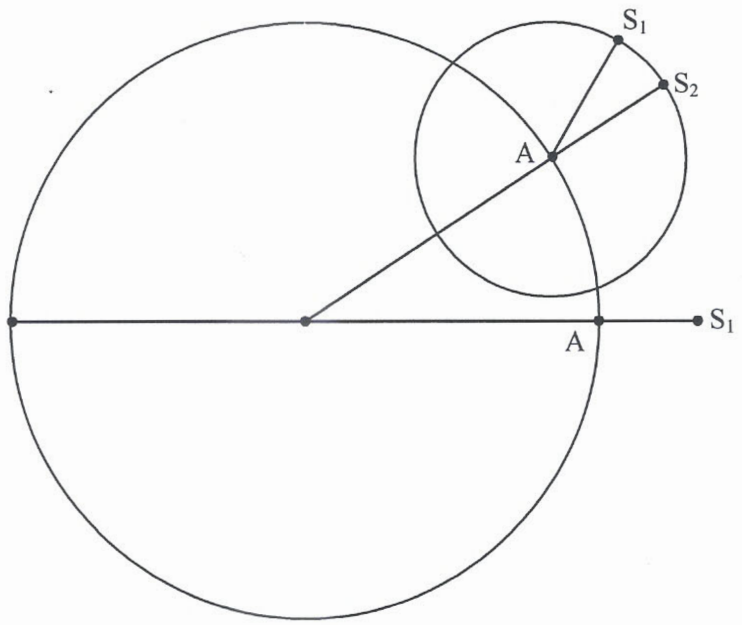


Fig: 3

The procedure is this: To get the mean Sun at Ujjain multiply the days from the Epoch by 800, deduct 442 and divide by 2, 92, 207. Then subtract 80° (*ucca* or *mandocca*). This is called *kendra* (*manda kendra* or mean anomaly). Multiply the R sine of the Sun by 14 (*manda paridhi*). Divide by 360° and get the arc. The arc is to be deducted from the mean Sun if the anomaly is less than 6 rās'is and added if it is greater than 6 rās'is.

In short, if m is the mean longitude, t is the true longitude, *ucca* is a and r is the *manda paridhi*, then $t = m - (\mathrm{R}\sin)^{-1}\left[\frac{r}{360°}\{\mathrm{R}\sin(m-a)\}\right]$.

In the case of the Moon the procedure is similar. For other planets namely Mars, Mercury, Jupiter, Venus and Saturn, there is also a *s'īghra* correction apart from the *manda* correction given above.

The procedure for getting mean positions are given. The *mandoccas* are available in the text. *S'īghra* for Mars, Jupiter and Saturn is the same as the mean Sun. For Mercury and Venus they are

separately given. The *s'īghra paridhis* and *manda paridhis* are given in the text. With these, we are in a position to compute the positions of star-planets.

Step I: Deduct the mean from the *s'īghra*. The remainder is *s'īghra kendra*. If it is less than 90°, 120 sin (*s'īghra kendra*) is called *bhuja* and 120 sin (90° − *s'īghra kendra*) is called *koṭi*.

If the *s'īghra kendra* is more than 90° and less than 180°, subtract it from 180°. If the *s'īghra kendra* is more than 180° and less than 270°, deduct 180° from *s'īghra kendra*. If it is more than 270° and less than 360°, deduct it from 360° and find *bhuja* and *koṭi* in all these cases.

The *bhuja* and *koṭi* must be multiplied by the planet's *s'īghra paridhi* and divided by 360°. Thus transformed, they are *bhuja* result and *koṭi* results related to the *s'īghra* correction. If the *s'īghra kendra* between 270° and 90°, the *koṭi* result is to be added to 120 (*trijyā*). If the *s'īghra kendra* is from 90° to 270°, the *koṭi* result has to be subtracted from 120. Square this and add to the square of the *bhuja* result and extract the square root and divide by that 120 × *bhuja* result. Find $(120\sin)^{-1}$ of this. Subtract half this from the longitude of *mandocca* if the *s'īghra kendra* is 0° to 180°. Otherwise add.

Step II: Half rectifying the *mandocca* position thus, deduct it from the mean. The result is to be used as *kendra* (anomaly) of *mandocca* in the second step. As we find the *bhuja* of *s'īghra kendra*, we find the *bhuja* of *manda kendra*. Multiply the *bhuja* of *manda paridhi* and divide by 360 ° and get the transformed *bhuja*-result for *mandocca*. Find $(120\sin)^{-1}$ of this. Add half of this arc to the half-rectified longitude of *mandocca* if the anomaly is from 0 to 180°, otherwise subtract.

Step III: Subtract this rectified *mandocca* from the mean and we get the *manda kendra*. Find the *bhuja* and multiply by the *manda paridhi* and divide by 360°. The *bhuja* result is got. Find $(120\sin)^{-1}$ of this and subtract from the mean if the *kendra* is between 0 and 180 °. Otherwise add. This is the rectified mean.

Step IV: Deduct the rectified mean from *s'īghra*. The *s'īghra kendra* is got. Find the *bhuja* and *koṭi* of this as in the Step I. Multiply the *bhuja* by *s'īghra paridhi* and divide by 360°. Multiply the *koṭi* by the *s'īghra kendra* and divide by 360 °. Following the procedure in Step I, and getting this result add to or subtract from 120 accordingly as the *kendra* is between 270° to 90°, or 90° to 270°. Square this and add to square of the *bhuja* result and find the square root. Divide *bhuja* result × 120 by this square root. Find $(120\sin)^{-1}$ of this. Add this to the rectified mean if *kendra* is from 0 to 180°. Otherwise, subtract. The geocentric longitude of the true planet is got. For Mercury and Venus additional procedure is given.

The method is similar to that given in various books on astronomy. But Varāhamihira suggests some corrections which require research.

Apart from the computation of planets various aspects of astronomy are discussed.

These give the general method of *Saurasiddhānta*. There is a later *Sūryasiddhānta*[9] which differs from the one discussed here. In that *trijyā* is taken to be 3438'. Also the variable epicycles imply the partial ellipticity of the orbits as shown by S. Madhavan[7].

From Varāhamihira's observation in *Bṛhatsaṁhitā* [B.S.17.1] it is believed that the present *Pañcasiddhāntikā* is not complete. It is also likely that Varāhamihira wrote a separate book on *Sūryasiddhānta*.

(ii) ***Bṛhatsaṁhitā***

Bṛhatsaṁhitā is encyclopaedic in its range and explains many physical phenomena scientifically. Varāhamihira gives the cause of eclipses thus:

bhūcchayāṁ svgrahaṇe bhāskaramarkagṛhe pravis'atīnduḥ |
pragrahaṇamataḥ pascānnendorbhānos'ca pūrvārdhe ||

B.S. 5.8

When the Moon enters the shadow of the earth, lunar eclipse is caused. When it enters the solar region, solar eclipse is caused. That is why a lunar eclipse does not begin at the west, nor does the solar eclipse commence at the east. The legend of Rāhu devouring the luminaries to retaliate for the unfair treatment given to him at the time of extraction of ambrosia from the milky ocean has been unceremoniously brushed aside.

While describing the rainbow he says,

sūryasya vividha varṇāḥ pavanena vighaṭṭi tāḥ karāḥ sābhre |
viyati dhanuḥsaṁsthāna ye dṛs'yante tadindradhanuḥ ||

B.S. 35.1

When the rays of the Sun of different colours are scattered and separated by the wind in the sky with clouds, the figure that appears is called a rainbow.

This suggests the refraction and reflection in the clouds (sheets) of particles, a rainbow is formed. It is similar to the modern theory though one cannot expect from an astronomer of the 6^{th} century A.D. to be more scientific than this.

Varāhamihira's contribution to combinatorics is also important. We get in *Bṛhatsaṁhitā* the following:

pūrveṇa pūrveṇa gatena yuktaṁ sthānaṁ vindruntyaṁ pravadanti saṁkhyām|
icchavikalpaiḥ kramas'oऽbinīya nīte nivṛttiḥ punarnya nītiḥ||

B.S. 77.22

This means that for getting nc_r, write the numbers 1, 2, 3,.......*n* in the direct order and in reverse order below and retain only according to the requirement (as desired). The result is

$$nc_r = \frac{n(n-1)....(n-r+1)}{1....2....r....}$$

(iii) ***Bṛhajjātaka***

One cannot expect mathematical ideas from this astrological work. But his definition of *paramocca* (deep exaltation, a technical term in Astrology) is this:

ajavṛṣabha mṛgāṅgana kulīrā
jhaṣavaṇjau ca divākarādituṅgāḥ |
das'as'ikhi manuyaktithīndriyāṁas'ai
strinavakaviṁs'atibhis'ca teऽstanīcāḥ ||

B.J. 1.13

According to this the *paramocca* of the Sun occurs in the 10^{th} degree of Aries. In fact it cannot be the whole of tenth degree. It is only a point which is at the end of the 10^{th} degree. According to the commentary *Apūrvārthapradarsi'kā*[1] on *Bṛhajjātaka* the metre *puṣpitāgrā*

(blossomed at the tip) suggests that it is at the end of the 10th degree but not equal to10°. 10° is the least upper bound. In those days when proper definition of points, lines and planes were not formulated, it is interesting that Varāhamihira has conceived of the concept of a point. In fact, Varāhamihira uses the same metre *puṣpitāgrā* whenever *paramocca* is referred to, corroborating the view of the commentator.

3. Conclusion

We have culled out from Varāhamihira's works, the scientific notions and mathematical principles used by him. By any standards he has a unique place. He was a poet and undoubtedly no other Indian astronomer had his versatility.

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Brahmagupta

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Bramha Gupta was the first mathematician who dared to correct some of the results of Arya Bhat (First) of fifth century AD. This article contains four parts: the life sketch of Brahma Gupta , the books he authored, his contributions to mathematics and comments on his work.

1.Life Sketch; Brahma Gupta was born in southern part of Rajasthan, a north-west state of India in 598 AD. He belonged to a well known 'Waghela' family of place Bhimnal . This place is situated between Mount Abu and river Luni (Latitude25 ^{0}, longitude72 ^{0}), close to Rajasthan-Gujarat border.

He was an authority on astronomy and astrology also. Because of high degree of intellect, he was appointed as chief of an observatory at a place Ujjain in the state Madhya Pradesh of India.

It seems that he was a disciple of another astronomer Varah Mihir from the same place Ujjain . He studied the works of Varah Mihir and made commentary on his works. It is said by many historians that Arya Bhat (I) stated the results of astronomy, but Brahma Gupta proved those results and also contributed some more.

2. Works of Brahma Gupta : Brahma Gupta wrote three texts (1) *Brahma Sphut Sidhanta* (various astronomical results of Brahma Gupta) *(2)Khand Khadyaka* (Sweet toast) *(3) Uttar Khand -Khadyaka* (More sweet toasts) All these texts are in Sanskrit Language.

2.1 Of all these texts, Brahmasphut- Sidhanta is supposed to be the standard and basic text in astronomy. This book contains 24 chapters and 1080 verses.

First ten chapters deal with astronomy. Supplements to these results are given chapters. 13 to 17.All these chapters include the motions of celestial bodies, their speed, solar and lunar eclipses- rise and decay of eclipses, lunar shades , predictions about full moon day and new moon day etc.

Chapter 11 contains criticism about the results of previous mathematicians and the corrections suggested by Brahma Gupta. This chapter is named as *Dushana* (criticism).
Chapters 12 and 18 contain arithmetic ,algebra and geometry... These include progressions, permutations, linear and quadratic polynomials, properties of cyclic quadrilaterals etc.

The 18 th chapter contains puzzles and various forms of quiz. This chapter is name as *kuttak* (puzzles, specially in algebra) chapter.

Previously arithmetic and algebra were termed as mathematics only. It was Brahma Gupta who separated arithmetic and algebra. Also he named algebra as kuttak branch of mathematics.

Chapters19 to 24 deal with solid geometry. These chapters include the volume, surface areas and other properties about cone, cylinder, sphere and other relevant results.
The 25th chapter, not included in this text is Dhyanagraha, containing 72 verses about astrology.

2.2 The second text, *Khand Khadyaka* (Sweet toast) contains chapters ,spread over 265 verses, all in Sanskrit language. It is a practical manual of Indian Astronomy. This text deals with time measurement

and about length of day, month and an year. Brahma Gupta claims that the length of a year is 365 days,15 hours and 30 seconds.

Many results about astrology are also mentioned in this book. The effects of movements of celestial bodies on human life and his nature is mentioned in *Khand Khadyaka* . This contains nine chapters, and expounds the midnight system.

Some historians commented that this text is meant to compete with similar results of Araya Bhat (I). but Brahma Gupta had refuted this charge and claimed that it is his original work.

This text , *Khand Khadyaka* became very popular then because it contained astronomy, as every one is eager to know about his future!.
It gives results useful in everyday life, birth, favorable timings of marriages etc. quickly and in a simple way.

In the Sindha province this text was translated in Sindhi language. This translation is named as *Sind-Hind*. This text is also known as *purva Khand-Khadyaka*

This text was translated into Arabic first by al-Fazari in 8th century and then by al-Biruni in 11 th century. It was because of this that Arabs were acquainted with a scientific system of learning and practicing astronomy. Bhaskarachaya(II), (11th century) ,described Brahma Gupta as " *Ganakchakrachudamani*" (Jewel among the ring of mathematicians)

2.3 The third text of Brahma Gupta is *Uttar Khand-Khadyaka* (More sweets). This text is a supplement to *Khand-Khadyaka*.It contains 5 chapters.

Uttar Khand-Khadyaka contains the numerical corrections to the results of the second text. Also it contains the various methods of proving the results and answers to some questions posed in the second text.

Brahma Gupta had a practice of writing numbers from right to left and also in the reverse way. He claimed that there are 20 operations like addition, subtraction etc.

He has given methods of finding the squares, square roots, cubes and cube roots of numbers. Some of these methods ore tedious and cumbersome, not applicable to all numbers , and hence were not popular.

Brahma Gupta proved that addition and subtraction, multiplication and division are inverse operations. He had given many examples for this verification.

3. Mathematical Contributions of Brahma Gupta:
1. Brahma Gupta's style of writing is compact and concise.
Brahma Gupta was aware about the concept and place value of zero, which came from Mayan mathematics (Central America). He stated the rules like

$$a + 0 = a,\ a.0 = 0,\quad a – 0 = a,\ a \times (-b) = -a \times b = -a \times b,$$

But he made a mistake by asserting that '0 divided by 0 is 1' . The only reason for this is that he could not foresee the logical complication of this absurd statement.

However, he did seem aware that division of a nonzero number by zero was a matter , because he did not offer any comments on the quantity

$$a \div 0$$

2. Method to find a square of a number:

Use the formula: $x^2 = (x + y)(x - y) + y^2$

Hence, $13^2 = (13+2)(13 - 2) + 2^2 = 15 \times 11 + 4 = 169$.

3. He stated the result (now known as Pythagoras theorem) in the following manner:

$(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$, where the sides of a right angle triangle are $(m^2 - n^2, 2mn$ and $m^2 + n^2)$

4. Following are two right angle triangles with hypotenuse 85 as :

$$75^2 + 40^2 = 85^2 = 51^2 + 68^2$$

Now draw a circle with diameter 85 . The two right angle triangles of sides (75, 40, 85) and (51, 68, 85) with hypotenuse 85 form a cyclic quadrilateral.

This is a method for drawing a cyclic quadrilateral.

5. Heron, (Egyptian mathematician, 50-100 AD.) obtained the formula for area of triangle as

Area of triangle = $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$,

Brahma Gupta extended this formula, 500 years later, but independently, for the area of a quadrilateral.

Consider a cyclic quadrilateral with sides a, b, c and d.

Let $2s = a + b + c + d$.

Then the area of this quadrilateral is given by

Area = $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$

Heron's formula is obtained by putting simply $d = 0$.

6. By actually calculating the volume of a sphere**,** Brahma Gupta has estimated the value of (pi) ð as 3.1543, which very close to its best approximation 3.14. He verified this result later by finding the area of a circle. Initially, he took $\pi = \sqrt{10}$

Brahma Gupta used to say that the "practical value' of π is 3, and 'neat value' is $\sqrt{10}$.

7. Consider the following interesting arithmetic puzzle given by Brahma Gupta.:

Find a number such that, when divided by 6 gives remainder 5,
when divided by 5 gives remainder 4, when divided by 4 gives remainder 3, when divided by 3 gives remainder 2, and , when divided by 2 gives remainder 1.

<u>Infinite</u> Answers given by him are 59, 59 + 60, 59 + 120, 59 + 180,
59 + 240, 59 + 300, and so on with a common difference of 60..
This common difference 60 is the least common multiple of all divisors.

Similar puzzles on cone, cylinder, sphere, and their frustums are given by Brahma Gupta with formulae, methods of solving these problems and their answers. One of them is the following:

A conical well has an opening of 10 square meters area and that of its base is 6 sq. mtrs. Its vertical height is 30 mtrs. Find the capacity of water it will contain.
He uses the formula: Volume of frustum to find the solution.

8. Brahma Gupta gave a telegraphic rule for the sine function of an angle as :
sine(0 , 30, 45, 60, 90) degrees = $\sqrt{\{[0,1,2,3,4]/4\}}$ respectively.

Similarly for other trigonometric functions. He derived the results about the spherical geometry also. These rules speak about high brilliance of Brahma Gupta and also about the high standard of mathematics in those days.

9.. Formula of lengths of diagonals of a cyclic quadrilateral:
Let a, b, c, and d be the sides of a cyclic quadrilateral.

Let x and y be the lengths of its diagonals.

Then, $x = \sqrt{\{(ad+bc).(ac+bd)/(ab+cd)\}}$,

and $y = \sqrt{\{(ab+cd).(ac+bd)/(ad+bc)\}}$,

10. A noteworthy contribution of Brahma Gupta is formation of cyclic quadrilaterals in which the sides, diagonals and area are all rational numbers and the diagonals intersect orthogonally.
The quadrilaterals are known as *Brahm Guptan Quadrilaterals*

He said : If there are two right angle triangles of sides (e, f, g) and (p, q, r) , the *Brahm Guptan Quadrilateral* will have the sides (e.r, f.r, g.p, g.q). Here, e.r stands for the product of e and r. etc.

11. Brahma Gupta's Lemma: This contains a conjecture to solve an equation $nx^2 + c = y^2$,where n is a natural number, This is similar to Pell's equation . Brahma Gupta solved this equation by recurrence relations.

Let α , β be the solutions of $nx^2 + c = y^2$ α_1 and β_1 be the solutions of $nx^2 + c_1 = y^2$ .Then $x = \alpha\beta_1 + \alpha_1\beta$ and $y = n\alpha\alpha_1 + \beta\beta_1$ are the solutions of the equation $nx^2 + cc_1 = y^2$ **.**

Interestingly, Brahma Gupta said that $x^2 - N y^2 = -1$ cannot be solved unless N is a sum of two squares.

Brahma Gupta proposed a problem of this type:
Solve: $x^2 - 92 y^2 = 1$, with his comment—"A person solving this problem, with integer solution, within an year is a mathematician !! "
The smallest solution Brahma Gupta gave is x = 1151 and y = 120

Using this theme, Bhaskaracharya (II),(11 th century), proposed a similar problem: $x^2 - 61 y^2 = 1$.
And the smallest solution given by him was
x = 1766319049 and y = 226153980.

(Interestingly, Archimedes (287-212 BCE) proposed a similar problem, about 700 years earlier,: Solve $\mathbf{x^2 - 4729494\, y^2 = 1}$.
And the smallest solution of this equation was that both x and y contain 206545 digits !!)

12. Interpolation. Prior to Brahma Gupta, the first order interpolation methods ,called linear interpolation, to find the intermediary functional values between the tabulated values, were known.

Brahma Gupta was the first to give the second order interpolation for equal as well as unequal tabulated intervals. This rule is equivalent to present Newton-Sterling interpolation formula.

13. Rational Box: Brahma Gupta's parameterization of rational triangles lead naturally to some interesting problems, One of the problems he proposed is of a 'rational box'

"Find a rectangular box such that the areas of each face is a rational number, all edges are rationals, the body diagonals and face diagonals are all rational numbers"

Many mathematicians, including Euler and Mordell tried for this box, but failed.
In 1719,Paul Halcke proposed the nearest solution:
The edges of this box are 44, 240 and 117, with integer face diagonals. However the body diagonals came out to be irrationals. The problem still remains unsolved.

14. The solution of a quadratic equation $ax^2 + bx = c$, given by Brahma Gupta is

$$x = \left\{-b/2 + \sqrt{\left[ac + (b/2)^2\right]}\right\}/\,a$$

which is same as $x = \left\{-b + \sqrt{\left[4ac + b^2\right]}\right\}/\,2a$ what we use now.

15. Rational triangles: Brahma Gupta defined a rational triangle as a triangle with rational sides and rational area., he asked to find such a triangle.

He proposed the following solution : A triangle with sides a, b, and c is a rational triangle if
$a = (u^2 + v^2) / v$, $b = (u^2 + w^2) / w$, $c = (u^2 - v^2) / v + (u^2 - w^2) / w$,
for some rationals u, v, and w.

A stronger claim, Brahma Gupta, is that and y such rational triangle can be split into two right triangles. These two triangles have the sides as

$[(u^2 - v^2) / v,\ 2u,\ (u^2 + v^2) / v]$ and $[(u^2 - w^2) / w,\ 2u,\ (u^2 + w^2) / w]$

Ex: One such rational triangle, Brahma Gupta proposes a problem, has sides 13, 14 and 15 Split this triangle in to two right angle triangles with rational sides.

The solution : The two right angle triangles have the sides
{5.12 and 13} and { 9,12 and 15}. the height of each triangle is 12, Their inclined sides are 13 and 15, and bases are 5 and 9 , so that their sum is 14. So that {13 , 14, 15} forms a rational triangle . Its area is 84.

Brahma Gupta stated the results about the rational isosceles triangle, rational trapezium, rational rectangle and irregular quadrilateral also.

16 . Brahma Gupta fairly accurately computed the circumference of earth .
17. Brahma Gupta stated the formulae for the length of common chord of two intersecting circles. Also, he stated the results for chords, segments, and areas they cover.

18. Brahma Gupta stated the formula for finding the side of any regular polygon inscribed in a circle of given diameter.

19. Brahma Gupta was the first to state that the volume of a cube is equivalent to volume of six regular tetrahedrons taken together.
At the same time he stated a formula for the volume of a tetrahedron.

20. Brahma Gupta had rules for calculating the time of the day from shadow- measurements, the lengths of shadow from the known heights of the gnomon and its horizontal distance from the light.

21. The rule for the radius R of circumcircle of a triangle is given by Brahma Gupta as:
$2R = a / \sin A = b / \sin B = c / \sin C$, where a, b, c, are the sides of a triangle ABC opposite to angles A, B, C respectively.

22. Brahma Gupta was first one to give the solution of Difointine equation $ax + by + = c$, where a, b, and c are integers. He stated that for the integral solution of this equation, the greatest common divisor of a and b should divide c.

He proposed that if a and b are relatively primes, then the solution of the above equation is
$x = p + m b$, $y = q - ma$, for some integers p, q and m is an arbitrary. integer.

4. Comments.
1. Besides mathematics and astronomy, Brahma Gupta had keen interest in Physics. He claimed the "bodies fall toward earth because of nature of earth to attract the body, as water flows along the slope of the surface." This indicates that he had a strong sense of gravitational force.

2. At the age of 30 yrs., he wrote the supreme text " *Brahma-sphut-sindhanta* " which was a miracle in the field of mathematics in those days.

3. All the period in his life, he was treated as' defeater of Aryabhat(I), and Brahma Gupta did not oppose to this comment. But in this second text Kand Khadyaka, he praised Arayabhat (I) for his wonderful results.

4.He was famous as astronomer and astrologer , though his work in mathematics was enough significant.

5. Brahma Gupta treated himself as disciple of and astronomer Varah Mihir (5 th century)

6. Brahma Gupta did not gave the proofs of all his results. Later on, these proofs were undertaken by the scholars of Aryabhat School like Bhaskara II.

7. Brahma Gupta did not give any expression about the area of trapezium, but mentioned many of its properties and its diagonals.

8. Brahma Gupta used laws of reflection to solve the problems of heights and distances.

9. Brahma Gupta used mathematics mainly to promote astronomy. That was practice in medieval India in those days.
10. Aryabhat (6th century) suggested the use of letters for the unknowns. Brahma Gupta used abbreviations for each of several unknowns in special problems. He also used suffixes in some cases.

11.A negative number was distinguished by a dot, and no bar is used to denote a fraction , as , — 3/ 4 was denoted by **.** 3 4.

12. Brahma Gupta discarded the negative root of a quadratic equation, and said that the square root of a negative number were absurd and hence did not exist.

13. An interesting comment on problems is given by Brahma Gupta
. He said **:** "The problems are proposed simply for pleasure; a wise man can invent thousands of such problems and also solve them;

As the Sun eclipses the stars by his brilliancy (sunlight), so a man of knowledge will eclipse the fame of others in the assembly of people, if he proposes algebraic problems and still solves them."

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Mathematical Inventions

Collection : Anant W. Vyawahare *, Nagpur

Introduction: This collection is meant to exhibit various mathematical results, in a comparison form. Many of the results , which we use, are attributed to foreign mathematicians. But looking through the pages of history of Mathematics, it observed that Indian mathematicians already obtained these results few hundred years earlier..

There is no intention to show which result is stronger or weaker, or any sense of superiority or inferiority. In the following, the (A) part of result is Indian invention, and immediate (B) part is a foreigner's claim. The period of each invention is also mentioned.

1 (A).Bodhayan: Diagonal square theorem (BC 800) .
The altars constructed for the worship are in rectangle / square shape , where the square of diagonal is equal to sum of squares of adjoining sides

(B) Pythagoras theorem ; (BC 540)

2. (A) Aryabhat's Kuttak by-ax = c (499 AD), where kuttak means algebraic equation Brahmagupta (6th century) calimed that this equation will have an integer solution providedgreates common divisor of a and b divides c.

(B) Diophantine Eq.(500 AD)

3.(A) Varahamihir ; Tri-Lostaka (488-587AD)
(B) .Pascal: Pascal triangle (1623-1662 AD)

4. (A) Brahmagupta $N x^2 + 1 = y^2$ (628 AD)
(B) John Pell :: Pell's equation (1610-1685 AD)
.
5.(A) Brahmagupta 's formula for area of cyclic quadrilateral

A= $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$ (628 AD)

(B) Heron's formula for area of a triangle

A = $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$ (100-50 BC)

6. (A) Virahank's series: 1,1,2,3,5,8,13,21........., (600AD)

(B) Fibonacci series 1,1,2,3,5,8,13,21...., (1170-1250 AD)

7. (A) Mahavira formula(850 AD) for combinations ^nCr = (n)! / (r!) (n-r)! (! stands for factorial)

(B) Herigone's formula(1634 AD)

8. (A) Bhaskaracharya (1114-1193) Formula for relative difference (retrograde motion)
(B) Rolle's theorem(1652-1719 AD)

9 (A).Madhav‘s theorem (1340-1425 AD)
$x = \tan x / 1 - \tan^3 x / 3 + \tan^5 x / 5 - ....$

(B) Gregory Series(1638-1675)

10. (A) Madhav‘s series II (pie) = 1-1/3 + 1/5 - 1/7 +...........(1340-1425 AD)

(B) Leibnitz ‘s expansion (1646-1716)

11.(A) Narayan Pandit (1356 AD) , factorization method

(B) Fermat‘s result (1601-65): divisiors of a number

12. (A) Bhaskaracharya (1114-1193 AD) method of finding greatest common divisor

(B) Euler's division algorithm (1707 - 83)

13 (A) .Permeshwara‘s formula for finding circum-radius of a cyclic quadrilateral (1360 AD)

(B) Huiler‘s similar formula (1782AD)

14. (A) Nilkanth Somyaji (1444-1545 AD), Summations "n, "n^2 and" n^3

(B) Euler‘s similar results (1707-1783 AD)

15 (A)Nilkanth Somyaji (1444-1545), sine rule: a / sin A =b / sin B = c / sin C

(B) Euler‘s results 2R = a / sin A =b / sin B = c / sin C,
where R is the circum-radius of a triangle ABC

16. (A) Brahmagupta(628 AD), volumes of frustum of cone and of pyramid

(B) Kepler (1571-1630, German)

17 (A) Jyeshtha Deo (1500 AD)
formulae for sin (x + y) and cos (x + y)in the text ‘*Yuktibhasha*‘

(B) Euler (1707-1783 AD)

18 (A)Jyeshtha Deo (1500 AD), Linear equations

(B) Liebnitz similar results (1646-1716)

19 (A)Jyeshtha Deo (1500 AD) volume and surface area of a sphere

(B)Liebnitz similar results by method of integration, (1646-1716)

20. (A)Shankar Variar (1500-60), Values of ð / 4,ð /16 in series
(B) Gauss similar results (1777-1855)

Comments:

1. This list is not all inclusive. More such results can be found out.

2 This list in not aimed at showing whose contribution is of higher intellect. It is an attempt to show that many of the results were obtained in India earlier.

3.. The concept and place value of ZERO is given by Maya civilization (250 -900 AD) of Central America (Mexico region).They used" a shell or an eye of a fish or bird " as a symbol for zero .

Zero should not be attributed to India. Varaha Mihir , a contemporary mathematician from India gave a symbol ' 0 ' ,(a circle), which is accepted universally.
For this clarification, it is not included in the above list.

4. Most of the ancient Indian literature in mathematics is in *Sanskrit* language or in *Modi* manuscript. Hence it was difficult to decode these inventions, and hence did not cross the Indian boundaries.

5. Foreign missionaries entered India through Kerala, where school of mathematics was academically rich. It is presumed that this knowledge of mathematics went from Kerala to Western countries and then were published there, hence their inventions were coined more than those of Indians.

6.It must be accepted that the results of the Western mathematicians are more stronger than those of Indians.

7.Lastly, the western mathematicians gave a 'written' proof of every result they invented; Indian mathematicians simply stated the results,(mostly without any proof), that too in story form , or in form of verses (for ex. 'Leelavati' of Bhaskaracharya).
Pythagoras theorem is mentioned is Shulba Sutra, but very rightly, the credit of this result goes to Pythagoras , because he 'proved' the theorem.

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Bhaskara - I (600 - 680 AD)

Sri V.G. UNKALKAR

The period 400-1200 AD is often known as golden age of Indian Mathematics. Mathematicians like *Aryabhatta, Varahamihir, Brahmagupta, Bhaskara – I, Mahaveer, Bhaskara – II* to name a few gave broader and clearer shape to many branches of mathematics. All these mathematical works were composed in Sanskrit in the form of *shlokas*(verses) or *sutras*(formulae) in order to memorize easily by students. This was followed by prose commentary by many scholars, so that the subject is thoroughly understood.

The most fundamental contribution of ancient India in mathematics is decimal system of enumerations including, of course, of zero. The world's most ancient literature – *Vedas* – contains a statement of 9 digits and zero and also huge quantities of arithmetic counting for which distinct numbers and designations existed in Vedic arithmetic unlike in our modern mathematics. The invention of zero expresses the total neutrality and the state of equilibrium of forces. It is the symbol of perfection and self completeness – symbolically quantified as small circle

Apart from the early contribution to the study of concept of zero as a number, negative numbers, so called modern definitions of sine and cosine were developed in India. These concepts were transmitted to Middle East, entire Europe and led further development that now form the foundations of many branches of mathematics.

The credit for the representation of numbers in a positional and systematic manner goes to the ancient mathematician *Bhaskara* (600 to 680 AD). He is commonly called as *Bhaskara I* to avoid confusion with 12th century mathematician *Bhaskara* (1114 to 1185 AD) who is known as *Bhaskara II*. *Bhaskara I* was the first mathematician to use first nine brahmi numbers (1 to 9) using a small circle for zero in a scientific manner in Sanskrit. Oftenly *Bhaskara I* explains a number given in this system using the formula '*Ankir Api*' which means 'In figure this reads' by repeating it in the written form. These figures are written from left to right in descending values as we write it today. Therefore, at least since 629 AD the decimal system definitely existed and known to Indians!

Much less is known regarding the life of *Bhaskara I*, but the clues to possible locations for his life such as capital of *Maitrika dynasty* of 7th century *Valabhi* (today's *Vale*), *Sivrajpura* – both in *Saurashtra* (in today's *Gujrat* State). Also mentioned are *Bharuch* (Broach) in South *Gujrat*, *Asmaka* (in *Andhra Pradesh*) and *Thanesar* (in *East Punjab*) which was ruled by king *Harsha* since 606 AD for 41 years. Hence the reasonable guess is that he was born in *Saurashtra* and later moved to *Asmaka* town. *Asmaka* is the location town of the famous astronomical school of followers of *Aryabhatta* I (475 to 550 AD). *Bhaskara I* is considered as the follower of *Aryabhatta I* and one of the most renowned scholars of Aryabhatta's astronomical school
Born in 600 AD in *Saurashtra,* his father educated him in astronomy and later he developed his astronomical and mathematical knowledge in Aryabhatta's astronomical school.

Bhaskara I wrote two treatises – The *Mahabhaskariya* (Great book of Bhaskara) and the *laghu Bhaskariya* (small book of Bhaskara). He also wrote commentaries on the work of Aryabhatta which is known as *Aryabhatiyabhashya*. This work was written in 629 AD and is known to be the oldest prose work in Sanskrit on mathematics and astronomy. All his contributions are mainly the expanded work of Aryabhatta and they were most popular in South India. *Bhaskara I* and *Brahmagupta* are the most renowned mathematicians who made considerable contributions to the study of fractions.

In his commentary on the work of *Aryabhatta*, *Bhaskar I* explains Aryabhatta's method of solving linear equations in detail and provides number of illustrative astronomical examples. He stresses in providing mathematical rules rather than just reifying on tradition or expediency.

Mahabhaskariya comprises of 8 chapters dealing with mathematical astronomy. The Verses deal with mathematics and contain variable equations and trigonometric formulae. This also includes solutions of indeterminate equation, rational approximation of sine function, and formula for calculating sine of an acute angle without use of tables correct to two decimal places. The formula in modern notation is

$$\text{Sin } x = 4x(180-x) \div [40500 - x(180-x)]$$

Where acute angle x in degrees, gives amazingly accurate value with less than 1% error.

He gave the relation between sine and cosine. He also provided the relation between sine of an angle greater than 90 to 360^0 to that less than 90^0.

He dealt with assertion: If P is a prime number, then 1+(P-1)! Is divisible by P. This was proved later by *Al Haithan.* It is also mentioned by *Fibonacci* and is now known as *Wilson's theorem*.

Bhaskara I stated a theorem about solutions of so called today's Pell equations. For example : As a problem he writes : '*Tell me, O mathematician, what is that square which multiplied by 8 becomes together with unity a square ?'* Speaking in modern terms he asked for the solutions of Pell equation

$$8x^2 + 1 = y^2$$

The solution being x = 1 and y = 3, from which further solutions can be constructed.
(eg x = 6 and y = 17)

Bhaskara I also discusses in both of his treatises, the chapters like planetary longitudes, heliacal rising and setting of planets, conjunction amongst planets, solar and lunar eclipses, phases of the moon etc. dealing with astronomical aspects.

His mathematical features include: numbers and symbolism, the classification of mathematics, the names and solution methods of equations of the first degree, quadratic equations, cubic equations and equations with more than one variable', symbolic algebra, unusual and special terms in Bhaskara's work, weights and measures, the *Euclidean algorithm* method of solving linear indeterminate equations, examples given by *Bhaskara I* illustrating Aryabhata I's rules, certain tables for solving an equation occurring in astronomy, and reference made by Bhaskara I to the works of earlier Indian mathematicians.

The mathematical world lost the great astronomer and great mathematician *Bhaskara I* in 680 AD at Asmaka at the age of about 80 years.

Viewing at the contributions of *Bhaskara I* in astronomy and mathematics, one can easily remember *Einstein*'s words," We owe a lot to Indians, who taught us how to count without which no worthwhile scientific discovery could have been made."

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Sridharacharya.Ac.750 A.D.

Prof. Virendra Kumar

Śrīdhara is the established Mathematician of the world. His name is remembered with great honour in the field of Mathematics.

उत्तरतो सुर निलयं दक्षिणतो मलय पर्वतं यावत्
प्राग परोदधि मध्ये नो गणकः श्रीधरादन्यः ।।
uttaratō sura nilayaṃ dakṣiṇatō malaya parvataṃ yāvat
prāga parōdadhi madhyē nō gaṇakaḥ śrīdharādanyaḥ ..

-quoted by Ballabha in the commentary on śrīdharā 's 'Trisatika'.

The method of solving a quadratic equation is famous with this name although it came in light by the books of other mathematicians as quotations. Sridhara was Hindu by religion. He is also known by the first verse of this 'Triśaṭika ' that Śrīdhara was devotee of Lord Śiva. Some persons believe that he accepted Jain religion in his later time.

There is a controvercy between different historians about the time and birth place of Śrīdhara. Some historians believe that Śrīdhara was a resident of Bengal while some others believe that Śrīdhara was a resident of Karṇāṭaka . Dr.K.S.Śukla does not agree with both opinions. Śrīdhara is mainly known as author of two Mathematical books named as-'Triśaṭika' and 'Pāṭigaṇita'. Three other books (i) Bījagaṇita (ii) Navasāti (iii) Bṛhaspati are also known by his name. These books are quoted by Bhāskara II (c.1150 A.D.), Makī Bhaṭṭa (c.1377 A.D.) and Rāghava Bhaṭṭa (c.1493 A.D.) respectively. One other book named as ' Gaṇitapañcavinsi ' also supposed to be written by sridhara but seeing its modern form Hayāsi does not believe it to be written by Śrīdhara. It has come to knowledge that one book of Nyāyaśāstra named 'Nyāya kandalī' was also written by sridhara who was resident of Karṇāṭaka . His father's name was Baldeva and that of his mother was Abbokā. Sudhākara Dvivedī writes that it was the tradition of our country that nobody else except astrologers used to give their names in their books.

In 'Nyāya kandalī' the name of author is given. It shows that the author of this book must be an astrologer. On this ground Sudhākara Dvivedī concluded that the author of 'Nyāya kandalī' and the author of 'Triśaṭika' both was same. Śrīdhara has also written two Astrological books 'Jatak Tilak' in Kannada and 'Jyotirjyānvidhi' in Sanskṛt. 'Nyāya kandalī' was written in Śak era 913. Therefore the time of Śrīdhara should be near about Śa k era 913. According to Sudhākara Dvivedī the time of Śrīdharawas c.991 A.D. After comparative study of Mahāvīra (c.250 A.D.) and Aryabhaṭṭa-II (c.950 A.D.) Dr.K.S.Shukta established the time of Śrīdhara between c.850 A.D. and c.950 A.D. some sentances of operation of 'Śrīdhara's book, are found in the book 'Gaṇitasāra Saṅgraha' of Mahāvīra which compel us to realize that Śrīdhara was born before Mahāvīra . Dr.Dutta and singh has opinion that the time of Śrīdharashould be about c.750A.D. Nothing can be said, with full confidence about the time and life history of Śrīdhara.

Mathematical work of Śrīdhara

There are two available books of Mathematics 'Triśaṭika' and 'Pāṭigaṇita' written by Śrīdhara. One copy of 'Trisatika' was in possession of Sudhākara Dvivedī's friend astrologer Rājājī and according to 'Gaṇakataraṅgiṇi ' its other copy was available in Karnataka state library. There are three hundred verses in this book. From the first verse of this book one comes to conclusion that this book is a summary of a larger book.

नत्वा शिवं स्व विरचित पाट्या गणितस्य सारमुद्धृत्य
लोक व्यवहाराय प्रवक्ष्यति श्रीधराचार्यः

natvā śivaṃ sva viracita pāṭyā gaṇitasya sāramuddhṛtya
lōka vyavahārāya pravakṣyati śrīdharācāryaḥ -'Triśaṭika'-1

After praying shiva sridharacarya is creating summary of arithmetic quoting his own written 'Pāṭigaṇita' for the sake of public.

This statement clearly, shows that Śrīdhara wrote another book on patiganita besides 'Triśaṭika' which mainly concerned with arithmetic. Acommentary of Śrīdharā 's Pāṭigaṇita is found in Kaśmīra which is edited and translated by K.S.Shukla in English. 'Pāṭigaṇita' is written in the form of prose. In the beginning of this book the tables of measurement of units are given. After this the methods of performing elementary operations of Arithmetic such as multiplication, division, square, cube, square root, cube root of numbers are given. Śrīdhara has given one or more solved examples for establishing rules of calculation of numbers, but he has not given their proofs. After the calculating rules of natural numbers Śrīdhara has given rules of rational fractions. He has given the questions related to proportions, elementary interest, purchase-sale, journey fairs, wage etc. In these examples some are very ordinary but supposed to be advanced. A portion of this book is related to arithmetical and geometric progressions. The end of this book comes with approximate hypothetical rules of area of some simple polygonal figures. But actually this cannot be taken as the end, because the next portion of this only available book is lost somewhere. The summary of the lost work of 'Pāṭigaṇita ' with its full contents is found in 'Triśaṭika'. Extra things which are not in 'Pāṭigaṇita' are topics related to volume, chiti, wood, rasi and shadow Mathematics.

Contributions and special features of Śrīdharā 's Mathematics

The special features of Śrīdharā's Mathematics which are valuable in the field of Mathematics are as follows:

1. Place names of numbers in decimal system – Arya Bhata has given the place names of numbers in decimal system as:

एक दश च शत च सहस्रम अयुत नियुते तथा प्रयुतम्
कोटयर्बुदं च वृन्दं स्थानास्थानं दशुगणम्

ēka daśa ca śata ca sahasrama ayuta niyutē tathā prayutam
kōṭayarbudaṃ ca vṛndaṃ sthānā sthānaṃ daśugaṇam - - Āryabhīya, Gaṇitapada2

The Arithmetic of Lalla, Brahmagupt, Baṭēśvar, Bhatta Balbhadra and Śrīpati are not available. Most probably they might have used these names in same form, but sridhara has given more names like this-

एकं दश शतमस्मात् सहस्रमयुतं ततः पर लक्षम्
प्रयुतं कोटिमथार्बुदं अब्जं खर्वं निखर्वं च
तस्मान्महा सरोज शङ्कुं सरितां पतिं ततस्त्वन्त्यम्
मध्यं परार्धमाहुयथोत्तरं दशगुणाः संज्ञाः

ēkaṃ daśa śatamasmāt sahasramayutaṃ tataḥ para lakṣam
prayutaṃ kōṭimathārbudaṃ abjaṃ kharvaṃ nikharvaṃ ca
tasmānmahā sarōja śaṅkuṃ saritāṃ patiṃ tatastvantyam
madhyaṃ parārdhamāhuryathōttaraṃ daśaguṇāḥ sañjñāḥ - Triśaṭika2,3.

Eka(1), Dasa(10), Sata(100), Sahasra(1000), Ayut(10000), lakṣa(100000), Prayut(1000000), Koti(10000000), Arbud(100000000), Abja(1000000000), Kharva(10000000000), Nikharva(100000000000), Mahasaroj(1000000000000), Sariku(10000000000000), Saritāpati(Samudra)(100000000000000), Aritya(1000000000000000), Madhya(10000000000000000), and Parardha(100000000000000000) these are the names of places of numbers of decimal system.

Bhāskara used these names as such. By the time of composition of the work of Śrīdhara, the decimal place value concept who well developed and generally known. This system has appeared amongst the Arabs in tenth century A.D. From there it went to Europe and it has now been adopted by all the civilized people of world.

2. Zero:-

Śrīdhara has given the definition and properties of zero in his book 'Triśaṭika'.

"If zero is added to any number, the number remains as such. If zero is subtracted from any number, the number remains unchanged. If zero is multiplied by any number, the result comes zero. If any number is multiplied by zero, the result is still zero. Similarly other operations are done on zero the result is zero".

Two things are clear from above statement-

(i) Ancient Hindu Mathematics knew the difference between axo and oxa although the result of the both is zero.

(ii) The meaning of other operations mean to divide zero by any number, squaring of zero, finding square root of zero, cubing of zero, finding cube root of zero etc. In the above explanation the division of any number by zero is not given any where. This shows that Śrīdhara knew that the division of any number by zero is not possible.

The Chinese(8^{th} century A.D.) left a gap or some vacant space between the numerals of numbers similar to the Babylonions where a zero was required. This is found on some Thang Ms.of the Tunhung cave temples. A symbol of a zero is usual circular form appeared only in c.1247 A.D. in a work sushu- chia- chang of Chin-Chin-Shio. The earliest use of zero in Arabia by a circular symbol was found in tenth century A.D. manuscript. The zero in its

circular symbol appeared in Europe in 13th century A.D. The Kaśmīria manuscript of the commentary on Śrīdhara 'Pāṭigaṇita ' used (.) as well as (o) for zero. The dot symbol was also found to have been used in Bhakhṣali Ms.(c.400 A.D.)

3. Use of (+) symbol:-

In the Bhakhṣali Ms., the modern plus sign(+) has been used to represent minus sign. Bhaskara-I has made use of a little circle (0) on the right of the number to be subtracted. For example $\left(\frac{1}{2}\right)-\left(\frac{1}{6}\right)$ was written as

$$\left.\underline{\frac{1}{2} \quad \frac{1}{6^0}}\right|$$

The plus sign to denote negative sign is also used in Jain work. 'Dhavata' a commentary on the ' Śatakhaṇḍagāma'. An other manuscript, a commentary on the 'Pāṭigaṇita ' of Śrīdhara available at kasmira has used the (+) symbol for minus sign. The symbol (+) is generally used after the number affected, but sometimes it was also used before the number affected. Thus -3 written as 3+ and some times as +3.

4. Use of Pati:-

The eight fundamental operations of Indian Arithmetic after the inversion of the decimal place value system of numeration are: Addition(Samkalita), Subtraction(Vyavakalita), Multiplication(Guṇana), Division(Bhāgahāra), Square(Varga), Square root(Vargamūla), Cube(Ghana), Cube root(Ghanamūla). These operations were carried out on a dust computing board(pātī). The method required to rubbing out of digits in every operation. According to Arabian scholars Al-Khwārizmī (c.825 A.D.) and Al-Uqlidisī (c.952 A.D.). Hindu Arithmetic entered Islam with dust abacus as an instrinstic tool of it. The later author replaced the dust abacuss to suit paper and ink. He used the Arabic word 'Takht' for 'Pātī'.

5. Multiplication of numbers

Śrīdhara has given four methods of multiplication of numbers. First is 'Kapāṭasandhi', second 'Tasth', third ' Rūpvibhāg' and fourth 'Sthānvibhāg'. Believing the popularity of the first one Āryabhaṭṭa has left this method in his book which was written in c.490 A.D. Due to popularity of this method Brhmagupta also left it in the chapter ' Gaṇitādhyāya' of his book ' Bṛāhmasphuṭa Siddhānta' which was written in c.628 A.D. Śrīdhara has given the name 'Kapāṭasandhi' to this method in his book 'Triśaṭika'. In the methods of multiplication of numbers first of all Bhaskara has given this method in his book
'Lilavati', but he has not given the name of it anywhere. Śrīdhara has given the name 'Pratyutpanna' to the product of two numbers. It seems to suggest that this method of 'Kapāṭasandhi' was known to India in 7th century A.D. This method reappeared in work of the Arabian scholars Al-Nasabi(c.1025 A.D.), Al-Hassar(c.1175 A.D.), Al-Kalasadi (c.1475 A.D.).

This method of Śrīdhara is as follows:

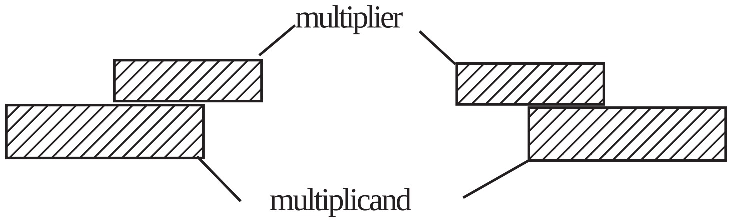
उत्सार्योत्साये ततः कपाट सन्धिर्भव वेदिदं करणम्

तस्मिंस्तिष्ठति यस्मात् प्रत्युत्पन्न स्तत्स्थः

utsāryōtsāhye tataḥ kapāṭa sandhirbhava vēdidaṃ karaṇam

tasmiṃstiṣṭhati yasmat pratyutpanna statsthaḥ - 'Trisatika'

Like in a closed door where one end of its board always covers the opposite end of the other board, the end digit of multiplier and the opposite end digit of multiplicand remain one upon other in starting stage. Hence this method is called 'Kapāṭasandhi'.



The main feature of this method of 'Kapāṭasandhi' was that, it was carried out in two stages:

(i) The relative positions of the multiplicand and the multiplier followed by.

(ii) The rubbing out of figures of the multiplicand and substitution of the figures of the product in their places.

Śrīdhara has explained the method 'Kapāṭasandhi' as follows-

'Put the multiplicand below the multiplier and multiply one by one directly or inversely and each time shift the multiplier by one place'.

Direct method of 'Kapāṭasandhi' is explained below with the help of an example.

Example: Multiply 69 by 245

Solution: Put 245 below 69 such that unit place number 5 of 245 remains under 6, the number of highest place of multiplier 69.

```
      6 9              ←
  2 4 5                6 9
                   2 4 3 4 5
```

Multiplying each digit of multiplier by unit digit 5 of multiplicand one by one. 5 x 9=45; putting 5 below 9 and 4 at any other place. 4 is in our hand and reserved. We shall use it at proper time. Multiply 5 by 6, we get 30. Add to it 4 of hand. We get 34. Replace the unit place number 5 of multiplicand by 34. Shift the multiplier right to left by one place.

```
    6 9                ←
  2 4 3 4 5            6 9
                   2 3 1 0 5
```

Below left number 6 of multiplier 69 is number 4. Multiply 4 by 9 we get 36. Now add it to second digit number 4 of the present number under multiplier. We get 40 and further replace 4 by 0. Now 4 is in our hand. Further multiplying 4 by 6. We get 24. Add to it 4 of hand and number 3 on the left of 4. Result is 31. Replace 3 by 1 and 4 by 3. Again shift the multiplier to left by one place.

```
  6 9                  6 9
  2 3 1 0 5            1 6 9 0 5
```

Proceeding in same manner we get the answer 16905.

Inverse Kapāṭasandhi Method

Multiplier and multiplicand are kept such that unit place digit of multiplier is on the highest place digit of multiplicand and at each step multiplier is pushed one place to right. We solve the above problem of multiplication as following.

I st step

```
  6 9              →
    2  4 5          6 9
                   1 3 8 4 5
```

II nd step

```
   6 9             →
1 3 8 4 5          6 9
                 1 6 5 6 5
```

III rd step

```
   6 9                6 9
1 6 5 6 5          1 6 9 0 5
```

16905 is product of 69 x 245.

In ancient India the operations were done on the board. Still now in some schools boards are used. Numbers in hand are written in the corner of the board. 'Pāṭigaṇita' is one of the names of arithmetic. In the above method other numbers are written on the places of some numbers after removing them. Therefore multiplication of numbers is called'Hanana' or 'Badha' in some ancient books. In above method multiplier is shifted to other place by one place at each step and multiplier is multiplied by the number of the multiplicand below in number of unit place / highest place of multiplier according to the method used.

6. Division

Śrīdhara has given the method of division. At that time partial divisions were done and partial quotients were written in separate line on the dust board and figures of these operations were obliterated. If the figures were not obliterated and the successive steps were written one below the other, this ancient process of division would become the modern method of long division. The method of extracting square root and cube root depends on the method of division.This seems to suggest that Arya Bhata and Brahmagupta knew methods of division. The method of division reappeared in the Arabian works from 9th century onwards. From Arabia, the method traveled to Europe where it came to be known as the gallery (galea, batello) method.

7. Method of finding square root and cube root of any number

In this 'Triśaṭika ' Śrīdhara has given the methods of finding square root and cube root of any number. They are in use till now Narayana Pandita has used the method of Śrīdhara in his 'Ganita Kaumudi'. In his book 'Triśaṭika ' Śrīdhara has given the method of determining the nearest square root of the numbers which are not perfect squares of any numbers. In sixteenth century A.D. both square root and cube root methods were given by Carteneo, which are exactly same as those of Arya Bhata I. Hence it is quite likely that the Indian methods of square root and cube root went to Europe through Arab intermediaries.

8. Fractions

Astrologers of old India felt difficult to work with fractions. But study of addition, subtraction, multiplication and other operations of fractions was well studied before Āryabhaṭṭa-I. Therefore Āryabhaṭṭa-I has omitted the study of fractions in the chapter 'Gaṇitapada' of his book ' Āryabhaṭīya. Āryabhaṭṭa has only given the squaring and cubing of fractions in his book. Brahmagupta and Śrīdhara have given all operations with fractions. In olden time the method of writing fractions was such that denominator was kept below numerator but there was no horizontal line between them. Śrīdhara has given six types of fractions and studied them. Śrīdhara has given the rules of simplifying the fractions to bring in their lowest form. Rules of their addition, subtraction and of other operations were given by him. We do not want to go into its details.

9. The Formulas for the sums of squares and cubes of numbers of A.P

Śrīdhara has given the formula for the sum of n natural numbers. He has also given the formulas for squares and cubes of natural numbers which form a A.P. Sridhara was first Mathematician who gave such type of formulas. The formula for lthe sum of squares of numbers of an A.P. is as follows:

द्विगुणित चमेन गणितं मुख सङ्गुणितं निरेक गच्छस्थ
कृति सङ्कलितेन युतं चमकृति गुणितेन वर्ग युतिः

dviguṇita camēna gaṇitaṃ mukha saṅguṇitaṃ nirēka gacchastha
kṛti saṅkalitēna yutaṃ camakṛti guṇitēna varga yutiḥ - 'Triśaṭika' 105

$a^2 + (a+d)^2 + (a+2d)^2 \ldots$ upto n terms

$[a + (a+2d) + (a+4d) + \ldots\text{..upto n terms}\ ]$

$+ d^2 [1^2 + 2^2 + 3^2 + 4^2 + \ldots\text{..upto n terms}]$

The formula for the sum of cubes of numbers of an A.P. is as follows:

श्रेढी फलस्य वर्गे प्रचयहते विहीन वदन गुणं
मूलफलावधि निदध्यादिष्टादि चयेन घन योगः

śrēḍhī phalasya vargē pracayahatē vihīna vadana guṇaṃ
mulaphalavadha nidadhyādiṣṭādi camēna ghana yōgaḥ 'Triśaṭika' 107

$+ a^3 + (a+d)^3 + (a+2d) + \ldots.upto$ n terms

$= S^2.d + S.a.(a-d)$ where $S = \frac{n}{2}[2a + (n-1)d]$

10. Śrīdharā 's Rational Solutions of Equation $Nx^2+1=y^2$:-

Historian K.S. Shukla studied the Śrīdharā 's method of finding integral solution of the equation of the type $Nx^2+1=y^2$ and found that the method used here is different from the method used by other Hindu Mathematicians.

11. Solution of Quadratic Equation

The most popular contribution of Śrīdhara to Mathematics is his method of solving quadratic equations. His book of Algebra is not available now but his method of solving Algebraic equation of second degree is quoted by some later Mathematicians. Bhaskara has given the formula of Śrīdhara in his book 'Bījagaṇita '.

चतुराहत वर्ग समै कृतैः पक्ष द्वयं गुणयेत
पूर्वव्यक्तस्य कृतैः समरूपाणि क्षिपेत्तयोरेव
caturāhata varga samai kṛtaiḥ pakṣa dvayaṃ guṇayēta
pūrvavyaktasya kṛtaiḥ samarūpāṇi kṣipēttayōrēva

The meaning of this is that multiply both sides of equation by four times of the coefficient of the second degree term of the unknown quantity. Further add the square of the coefficient of unknown quantity to both sides.

Śrīdharā 's method of finding the solutionof a quadratic equation is like this:-

Let the equation of second degree be $ax^2+bx=c$.

Multiplying both sides of equation by 4a we get $4a^2x^2+4abx=4abc$

Adding b^2 both sides - $4a^2x^2+4abx+b^2=4ac+b^2$

This can be written as $(2ax+b)^2=b^2+4ac$

Taking square root of both sides

$$2ax+b=\pm\sqrt{b^2+4ac}$$

we get $x=\dfrac{-b\pm\sqrt{b^2+4ac}}{2a}$

This method is in use of school students till now. Using this formula Śrīdhara has also given the formula for finding out the number of terms of a arithmetical progression whose first term, common difference and sum is given.

If 'a' is the first term, 'd' common difference and 'n' the number of terms of a arithmetical progression. The sum 's' of it is given as-

$$\frac{n}{2}\left[2a+(n-1)d\right]$$

i.e., $n^2d+n(2a-d)=2S$

using Śrīdharā 's formula we get,

$$n=\frac{d-2a\pm\sqrt{(2a-d)^2+8dS}}{2d}$$

Many other mathematicians before Śrīdhara used this formula. This shows that our ancesters also knew the Algebraic method of solving quadratic equations. But Śrīdhara was the first who directly gave this method. Seeing the importance of Śrīdharā 's Mathematical work several commentaries are written on his 'Triśaṭika' and 'Pāṭigaṇita '. The beginning verse of this article is found as a quotation in Ballabha's commentary of 'Triśaṭika'. Śrīdhara is immortal in Mathematical world.

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Lallacharya – An Unknown Astronomer & Mathematician

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Lallacharya was one of the leading & eminent Indian mathematicians, astronomers & astrologers of the eight century. This paper contains life & work of Lalla.

1) Life of Lallacharya (720A.D.-790A.D.)

Lalla was the son of Trivikarma Bhatt & Grandson of Taladhvaja. His date of birth is not mentioned any where ,so it is determined on the basis of some evidences. Amaraja's commentary on the *Khadakhadyaka* of Brahamagupta mentions Lalla as a commentator of that work.

The main evidence in support of Lalla's date can be decided as follows:

'Subtract 420 from the Saka year elapsed. Multiply the remainder severally by 25, 114, 96 , 47 & 153. Divide each product by 250. The quotients in minutes should be subtracted respectively from the mean longitude of Moon, its apogee & node , Jupiter & the *Sighrocca* (Sighra apogee) of Venus. Again multiply the above remainder severally by 48, 20 & 420. Divide each product by 250. Add the quotients in minutes respectively to the mean longitude of Mars, Saturn & Sighrocca of Mercury.'

What could 250 signify? It shows that Lalla corrected the positions of the planets calculated with the constants of Aryabhata's, by his own observations. According to the formula, 250 years after 420 Saka , which is Aryabhata's time , or, in 670 Saka (A.D.748) . Thus A.D.748 may well denote Lalla's time.

There is much similarity between Lalla's SD (*Shishyadhividhidantantra)* & Brahmagupta's *Brahmasphutasiddhanta*. He was a follower of Aryabhatta, Brahmagupta & Bhaskara, hence he is considered as the mathematician of 8th century , believed to have been born in 720 AD & died around the year 790 AD.

2) What part of the India did Lallacharya belong to?

Lalla compares the half moon with the forehead of place Latadesa, which is the south part of state Gujarat. Again in the *Ratnakosa*, in the section on seasons called *Rutucarca*, he says(in Sanskrit language) "Lalate latinam luthitam alakam tandavayati." Means wisps of hair play havoc with the foreheads of the women of Lata.

Lata was well known in the Deccan region and is mentioned in the history of these parts. On the basis of this fact Lalla appears to have belonged to place Latadesa.

3) Work of Lallacharya :

Lalla's notable works (all in Sanskrit language) include.

1) *Shishyadhividhidantantra* – A work on astronomy in two volumes.
2) *Siddhantatilaka* – Another work on astronomy.
3) *Rathnakosa* – A work on astrology, available in manuscript.
4) Astrological work.
5) Mathematical work

4) ***Shishyadhividhidatantra*** **i**s the only work on astronomy by Lalla which is at present available.

'*Shishyadhividhidatantra*' means a treatise that increases the knowledge & intellect of students of astronomy. . It is available in two volumes 'Grahadhaya' & 'Goldadhyaya'

The first volume '*Grahadhaya*" explains computation of the positions of the planets which includes several assumptions of Aryabhata. It has thirteen chapters on Mean planets, True Sun & Moon ,True planets, Three problems, Lunar & solar eclipses, Possibility of an eclipse

It includes longitudes of the planets, Rising & setting of the planets, Cusps of the moon, Conjunction of planets, Conjunction of the planets with the stars ,*Vyatipata* (Time when the sum of the longitudes of Sun & Moon is 6

signs)& *Vaidhrta* (Time when the sum of the longitudes of Sun & Moon is 12 signs) & rationale of Corrections. The thirteenth chapter is also called *Uttaradhikara* .

Second volume "*Goladhyaya*" concentrates on the sphere related computations & principles applied in astronomy .It contains nine chapters on graphical representation of motion of planets construction of the armillary sphere , Rationale of rules , Mean motion , Sphere of earth , Motion of the celestial sphere , Description of the earth , False notion , Astronomical instruments & Astronomical problems.

4.1)*In* ***Shishyadhividhidatantra*** Lalla uses the astronomical parameters of the Aryabhatta but his work is more comprehensive & explanatory. He also gave corrections to Aryabhata's results. This creation influenced even Bhaskara II and a commentary is written by him.

Lalla uses Aryabhata's constants with one significant difference. What Aryabhata calls the number of rotations of the earth, Lalla terms as the number of rotations of the celestial sphere or the sphere of constellations around the earth.

Lalla borrows the table of R sines from Aryabhatta, using 3438 as R (*R = 3438, the number of minutes in a radian to the nearest integer) and* dividing the quadrant into arcs of 225' each. But in the last chapter he gives another table called *Laghujya*. There he uses 150 as R & divides the quadrant into arcs of 10^0.

Some of the methods of astronomical calculations are of the same as those in the *Aryabhatiya* e.g. determination of the true places of the Sun ,Moon & planets , use of spherical trigonometry to obtain declination , earth –sine, calculation of obscured portion in a solar or lunar eclipse etc.

Lalla uses R versed sine for determining *ayana- valana* (Deflection of ecliptic from the equator on horizon of eclipsed body) , *ayanadrkkarmasu* (visibility correction) and the illuminated portion of the moon. He does not give any formula for the illuminated portion of the moon. Bhaskara I uses R versed sine in all the three cases in both the *Mahabhaskariya* & *Laghubhaskariya*.

A historically important point of observation regarding the SD is that so many of the special rules here are found in *Brahmasphutasiddhanta*. Calculations of *suddh*i (mean heliocentric position for superior planets , mean position of Sun), *laghvahargana* (Civil says between end of one solar year & any date in current year) & hence the mean longitudes of the Sun , Moon etc ,

Applications of parallax in a solar eclipse , correct time of conjunction of planets & that of pata (Moon's node; time when sum of Sun's & Moon's longitudes is 6 or 12 signs), are only few such cases.

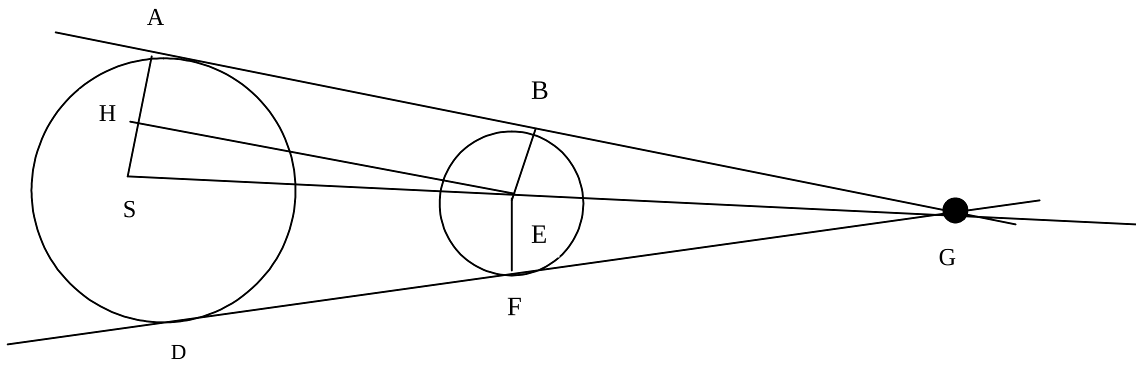
Lalla gives special methods to calculate the correct true places of Mercury & Venus. Lalla in all probably borrowed Brahamagupta's special rules & incorporated them in his work. It is a good synthesis of Arayabhata's & Brahmagupta's work.

Bhaskara II wrote a commentary on the SD, now available in manuscript form. He also has borrowed profusely from Lalla. He, however, does not hesitate to criticize Lalla in his *Siddhantasironani*, wherever the latter went wrong. Some of the criticisms are very correct e.g. Lalla's formula for the true motion of a planet; his use of R versed sine for calculating *valana* (Deflection) & correction, Goladhyaya, being the second section of SD is of great historical significance.

It contains the chapter on sphere of earth & description of earth, which gives the idea of geography of that time . The chapter on 'False notions' acquaints the readers with the current astronomical beliefs. The chapter on 'Astronomical instruments' gives a glimpse of the practical methods used for observations.

Lalla is the first astronomer whose work on astronomy contains a section on the 'Celestial sphere' also which is considered as Lalla's originality.

4.2)Method to calculate the height of the cone of the earth's shadow during Lunar Eclipse.



Let S be the Sun, E the earth AB & DF their common tangents meeting in G. EH is parallel to AB

GE is the height of the cone of the earth's shadow or *mahiprabha*

△ GAS ~ △ GBE –A A TEST.
△ GAS ~ △ EHS ---AA TEST
△ GBE ~ △ EHS

$$\frac{GB}{EH} = \frac{BE}{HS} = \frac{GE}{ES}$$

$$GE = \frac{ES \times BE}{HS} = \frac{ES \times BE}{AS - AH}$$

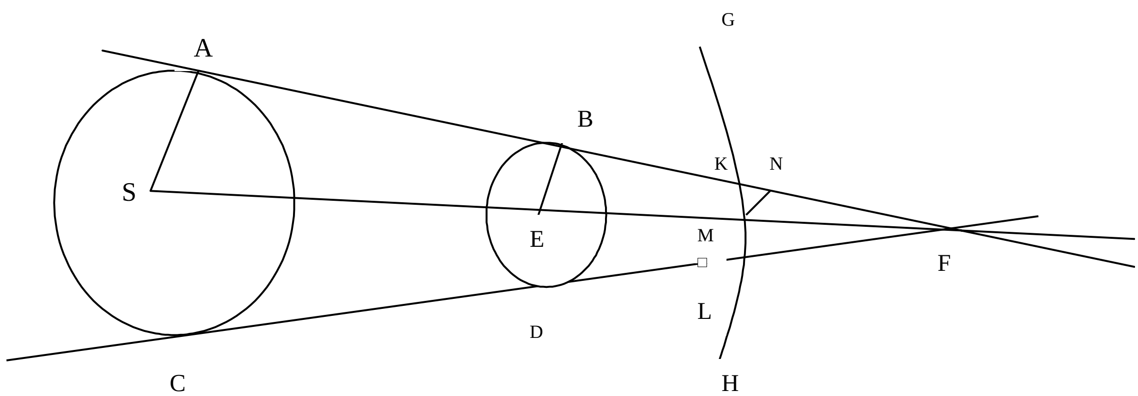
$$= \frac{Sun's\ distance\ from\ earth\ x\ earth's\ radius}{Sun's\ radius - earth's\ radius}$$

$$= \frac{Sun's\ distance\ from\ earth \times \frac{1050}{2}}{\frac{4410 - 1050}{2}}$$

$$= \frac{Sun's\ distance\ from\ earth \times 525}{1680}$$

$$= \frac{Suns\ distance\ from\ earth \times 5}{16}$$

4.3) Method to find the diameter of the earth's shadow during Lunar Eclipse .



Let S be the sun, E the earth AB & CD the common tangents meeting at F, GH the Moon's orbit meeting the tangents in K & L, and SE produced in M.

Then KML is the path of the moon which is within the shadow.

$Let\ \ MN\ \perp ABF$

Then the diameter of the earths shadow within the moons path is measured by 2MN approximately.

$\triangle\ \ FBE\ \sim\ \triangle FNM$ -- AA TEST

$$\frac{FB}{FN}=\frac{BE}{NM}=\frac{FE}{FM}$$

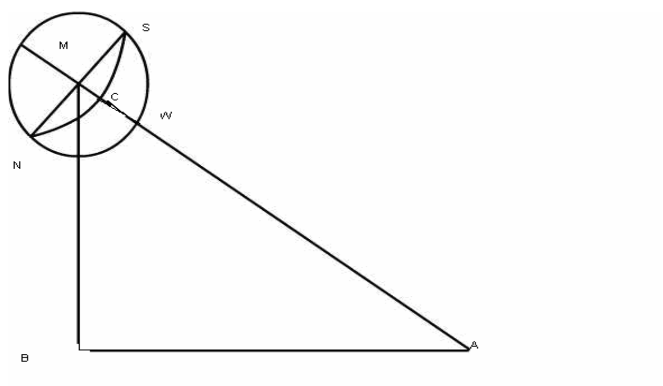
$$Now\ MN=\frac{EB\times FM}{FE}$$

$$MN=\frac{Earths\ \ radius\ (\ FE-EM)}{FE}$$

$$2MN=\frac{1050(earthsShadow-moon'sdis\tan ce)}{Earth'sshadow}$$

4.3) Method to show the cusps of the moon

E



Let A be the sun. AB is the *bhuja* (segment) drawn north, if it is north, or south if it is south. AB is then a north –south line on the plane surface.BM is drawn perpendicular to AB at B & equal to the koti or perpendicular. It is drawn to the west if the Moon is in the eastern hemisphere and to the east , if the moon is in the western hemisphere.

AM is joined & is the *Karna* or hypotenuse. Then M is the centre of the Moon's disc. Let AM cut the disc at W & E when produced. Then EW is the east-west line in the Moon's disc. Draw NS at right angles to WE through M. Then NS is the north-south line.

Now WE is the line along which the breadth of the illuminated or dark portion is to be marked. This breadth should be marked from

W , the west point , along WE, if H is the breadth of the illuminated portion for the bright half of the lunar month. But if the illuminated portion, is for the dark half of the lunar month , then it should be marked in the reverse manner.

Let WC be the portion. Then a circle is drawn passing through N,C & S. Thus SWNC is illuminated portion of the Moon's disc.

Bhaskara I's method are same that of Lalla. Bhaskara II's method are improved methods.N is the lower cusp of the Moon & is on the side of AB , the bhuja. When the illuminated portion is equal to half of the Moon's disc which is bisected by the diameter, the Moon has the beauty of the forehead of a woman belonging to the Lata- desa.

4.4) **Possibility of an eclipse:**

In a lunar eclipse the moon is at a distance of 6 signs from the sun . So if the difference between the longitudes of the sun and the moon's node is 6 signs, the moon must coincide with its node & consequently, its latitude is 0, so there is the possibility of a lunar eclipse.

In a solar eclipse, the distance between the Sun's & Moon's place is zero or 12 signs. So if the difference between the longitudes of the sun & Moon's node is 12 signs, the Moon must coincide with its node & consequently its latitude is 0. So there is the possibility of a solar eclipse.

According to Lalla, the sum of the radii of the moon & the earth's shadow at their mean distance is 56'. So there is the possibility of a lunar eclipse, if the latitude of the moon is less than 56'.. This will be so if the difference between the longitudes of the Moon & its mode is within 12^0.

The sum of the radii of the moon & the sun at their mean distances is 32'.. So there is a possibility of a solar eclipse if the latitude of the moon is less than 32'. This will be so of the difference between the longitude of the moon and its node is within 7^0.

4.5 Conversion of minutes to anguals:

The relation taken by Lalla is $\boldsymbol{2\ \frac{1'}{3} = 1\ angula}$

(1 angula =16mm to 21mm)

Sun's angular diameter = $\frac{11}{20} x$ true motion in minutes.

$$= \frac{11}{20} \times \frac{3}{7} \text{ True motion in angula}$$

Moons angular diameter

$$= \frac{11}{272} x \text{ True motion in minutes}$$

$$= \frac{11}{272} \times \frac{3}{7} \text{ true motion in angular}$$

4.6 Probability of an eclipse at any time:

i) If one wants to know whether there will be an eclipse after 6 months following steps can be followed.

1) First find the mean longitude of the sun, Moon, its apogee and node on that day .

2) Then add it to the first three longitudes $5^s, 24^0 27'. 6''$, $5^s 22^0 12'53''$, $0^s 19^0 42'53''$ and subtract $0^s 90^0 22'41''$ respectively from the longitude of the node.

3) The process must be reversed to determine the eclipses of the previous 6^{th} months.

4.7 Possibility of *Vyatipata* & *Vaidhrata*

When the sum of the true longitudes of the Sun & Moon is 12 signs & they are in different hemispheres of the ecliptic but on the same side of the solistitial points, there is *Vaidhrtayoga*. But when the sum is 6 signs & they are in the same hemisphere of the ecliptic but on the opposite sides of the solstitial points, there is *Vyatipatayoga*.

तुल्येश्प्यने भिन्नदिशो रवीन्द्वोः स्याद वैधश्तश्चक्रसमे समासे
भिन्नेश्प्यने तुल्यदिशोस्तु योगे चक्रार्धतुल्ये व्यतिपातयोगः

This verse defines *Vyatipatayoga* & *Vaidhrtayoga*. The mean declination or *madhyamakranti* (Declination of place of planet on ecliptic)of the moon is that declination which is calculated from its longitude , and hence it is the declination of the position of the Moon on the ecliptic.

Its true declination or *sphutakranti*(Declination of planet's centre) is the sum or difference of its *madhyamakranti* (Declination of place of planet on ecliptic)& latitude of viksepa(Celestial latitude of planet).The Sun has no latitude & so its *madhyamakranti* is its *sphutakranti* or briefly kranti

According to the Indian astronomers, that whole period is called *vyatipatakala,* during which the sum of the true longitudes of the Sun & Moon is somewhere near 6 signs & the true declination of any point on the disc of the Moon is equal both in magnitudes & in direction to the declination of any point on the disc of the Sun.

Vaidhrtakala is that period during which the sum of the true longitudes of the Sun & the Moon is somewhere near 12 signs & the true declination of any point on the disc of the Moon is equal in magnitude but opposite in direction to the declination of any point on the disc of the Sun.Both *Vyatipata* (Time when the sum of the longitudes of Sun & Moon is 6 signs)& *Vaidhrta*(Time when the sum of the longitudes of Sun & Moon is 12 signs) are denoted by *Pata.*

4.8) False Notions :

In this chapter Lalla mentions some of the beliefs prevailing then with regard to astronomical phenomena & refutes them. Most of these beliefs occur in the *Puranas.*

The views are as follows:

1) The days of the observers at the north pole begins when the Sun starts for the summer solstitial point & that of the observers at the south pole begins, when it starts for the winter solstitial point.(solstitial is that great circle of the celestial sphere through the celestial poles and the solstices)
2) The night comes when the mountain *Meru* covers the Sun.
3) Directions can be determined at Meru.
4) The dark half of the lunar month is the day of the manes & the light half is their night.
5) The Moon's orbit is above that of the Sun.
6) The illuminated portion of the Moon decreases because it is being sucked by the gods.
7) The earth is infinite.
8) The earth is plane like a mirror.
9) The earth moves.
10) The earth is supported in various ways.

Lallacharya refutes them all, as follows:

1) Lalla's argument is that if the observers at the North Pole can see it descending from Aries to Gemini, they should also see it descending from Cancer to Virgo when the Sun travels exactly along the same diurnal circles. So the current belief cannot be correct.
2) Lalla maintains that the night is not caused by *Meru* but by the shadow of the earth.
3) Lalla says that no direction can be determined at *Meru* , because there the observer's horizon coincides with the celestial equator & hence there is no east point.
4) Lalla states that the whole of the light half of the lunar month cannot be the day of the manes, because they do not see the Sun after the eighth day.
5) Lalla says that if the Moon were above the Sun , it would always be illuminated like a star. Moreover then it could neither cause a solar eclipse nor could it be obscured by the earth's shadow.
6) If the Moon's daily decrease were due to its being sucked by the gods mathematics would be of no use in computing its light & dark portions.

7) Lalla says that the earth could not be infinitely large , as then the sphere of the fixed stars could not go round it in one day.

8) Lalla maintains that the earth is spherical & not a plane. But as only a small portion of it is visible at a time , that may be the reason for its appearing as level as a mirror. If it were level , the tops of high trees could be seen even from a great distance.

9) The Bauddhas maintain that the earth was falling down in space. So Lalla says that if it were so, how could a thing when thrown up come down again on the same piece of a ground. Again ,if the earth were continuously moving up, the constellations would be nearer every moment.

Neither could the earth be moving from east to west nor from west to east. If it did , the birds would not be able to find their nests.Lalla tries to refute Aryabhata's theory that the earth rotates from west to east.

10) He refutes the belief that the earth is supported by an external agency.Lalla says that the earth remains unsupported, suspended in space. If it were supported by something, the latter , in its turn , would have to be supported by something else & so on. Then , there would be no end of supporters ,& that is not possible. Many wonderful things happen in this world ; so there is nothing to be surprised at if the earth hangs in space.

5) *Siddhantatilaka* : This is another work of Lalla on astronomy which is mentioned by Yallaya . This contains synthesis of Aryabhata's Aryasiddhanta & Brahmagupta's *Brahmasphutasiddhanta*. In this Lalla gave large number of methods to calculate the mean longitudes of planets.

Here Lalla said that God Brahama was born 26, 56,800 crore years back.

6) *Ratnakosa* : It is a fair sized work with astrological topics available in manuscript. It contains

1) The effects of tithes, *Karanas* (Time during which moon gains 6^0 over the sun), week days, *muhurtas* & stars.
2) Different stages of child birth.
3) Details of house building, temple building, setting up of idols, sowing plants.
4) The good & bad features of a man & woman.
5) Times for journey.
6) Types of elephants & horses to be bought.
7) The signs of rain.

The Historically important points to note are:

1) The words *Kuhu & Sinivali*, are the names of the new moon day & the day before that, respectively used in *Vedic* literature also occur here.
2) *Madhu*, *Madhava & Nabhasya* are used as names of months, the same in the *Vedic* literature.
3) *Atri*, *Garga*, *Parasara and Vasistha* have been named as ' Munis' or 'sages'.

7) Work on Mathematics

Lalla must have written on mathematics ,though no such work is at present available .From the comments of Bhaskara –II in *Goladhyaya* he quotes that Lalla as saying in his *Ganita* that area of the surface of a sphere is 2IIr. IIr^2 then contradicts him.

Even Narayan in his text *Ganitakumudi* says that Lalla's formula for the length of the hypotenuse of a quadrilateral is wrong. Lalla has given formulae on Geometry, Algebra & Indices. He mentioned that height of the atmosphere is 12 Yojana(Vedic measure of distance used in ancient India. The exact measurement is disputed amongst scholars with distances being given between 6 to 15 kilometers (4 to 9 miles).

Astrology of that time was based on astronomical tables & often the horoscopes allow one to identify the tables used. Some Arabic horoscopes were based on astronomical tables calculated in India.

The most frequently used tables were by Artabhata I. Lalla improved on these tables & he produced a set of corrections for the moon's longitude. One aspect of Aryabhata I's work which Lalla did follow was his value of

pi=ð $= \frac{62832}{20000}$ =3.1416 which is a correct to the fourth decimal place.

Astronomy cannot be studied without mathematics & hence his work on mathematics cannot be ignored.

Though the great Lallla shows influence of Aryabhata, Brahamagupta & Bhaskara ,his work was more systematic. He emphasizes on corrections & rejections of flawed principals. His work was later followed by astronomers like Sripali, Vatesvara & Bhaskara II.

Number of Words: 3314

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Ganit- Sar-Sangraha of Mahaaveeracharya

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This article deals with the pioneer work of **Mahaaveeracharya** (**Mahaaveera**) of 9th century. It is said that he had obtained surprising results in Mathematics in those days when no mechanical tools were available. Whatever he had done shows his talent. He had written a nice book of Mathematics known as गणितसारसंग्रह **Ganit-Sar-Sangraha** (**GSS**).

The great Mathematician **Mahaaveeracharya** was born in 800 AD at Mysore in Karnataka. The Jains have always been very interested in Mathematics. One distinguished Jain mathematician monk was **Mahaaveera** who wrote **GSS** in 850 AD during the reign of the great Rashtrakuta king Amoghavarsha. Amoghavarsha had become a Jain monk in the later part of his life. His capital was in Manyakheta in modern Karnataka. **Mahaaveera** died in 870 AD. **Mahaaveera** is the third name in the hierarchy of Bharatiya Mathematicians in this era. The first two are **Aryabhata** (499 AD) and **Brahmagupta** (626 AD). The **Aryabhatiya** of Aryabhata and **Brahmasfutsiddhanta** of Brahmagupta contain a little bit of Mathematics and more of Astronomy but **GSS** contains only Mathematics. **Mahaaveera** knew very well about these two. On going through **GSS,** three best qualities: the first one is Mathematician's intellectual sharpness, the second one is a poet's imagination and the third one is an artist's creativity are visible in **Mahaaveera**. Filliozat has written that the **GSS** deals with the teaching of Brahmagupta but contains both simplifications and additional information. Although like all Bharatiya versified texts, it is extremely condensed. The work of **Mahaaveera** from a pedagogical point of view has a significant advantage over earlier texts. It consisted of nine chapters and included all mathematical knowledge of mid-ninth century Bharat. It provides us with the bulk of knowledge which we have of Jaina mathematics and it can be seen as in some sense providing an account of the work of those who developed this mathematics. There were many Bharatiya mathematicians before the time of **Mahaaveera** but, perhaps surprisingly, their work on mathematics is always contained in texts which discuss other topics such as astronomy. The ***GSS*** is the earliest Bharatiya text which we possess which is devoted entirely to mathematics. In the introduction of the work; **Mahaaveera** paid tribute to the mathematicians whose work formed the basis of his book. These mathematicians included Aryabhata I, Bhaskara I and Brahmagupta. **Mahaaveera** had said that with the help of the accomplished holy sages who are worthy to be worshipped by the lords of the world; I glean from the great ocean of the knowledge of numbers a little bit of its essence in the manner in which gems are picked from the sea, gold from the stony rock and the pearl from the oyster shell and I give out according to the power of my intelligence, the **GSS**, a small work on arithmetic which is however not small in importance.

Mahaaveera had declared and emphasized about the universal utility of Mathematics by saying:

"*बहुभिर्प्रलापैः किं त्रैलोक्ये सचराचरें यत्किंचिद्वस्तु तत्सर्वं गणितेन विना नहिं* "

[GSS Chapter 1 Verse 16**]**

It means "*Whatever exists in this visible and invisible, movable and non-movable world; is only due to Measurement and computations in other words Mathematics*".

GSS contains nine Chapters. All are in *Sanskrit* language. These Chapters contain elementary arithmetic operations, measuring units, measurement of weight and length of gold-silver ornaments, area and volume of various bodies etc. Elementary algebraic equations of one variable were used for distribution of property, purchase-sale transactions etc. The Chapters are named by the topic it contains. The Chapterwise details of **GSS** are as follows:

Chapter 1: संज्ञा अधिकारः *Terminology:* In this Chapter; measuring units for line, time, grains, silver-gold and land are defined. Elementary mathematical operations like addition, subtraction, multiplication and division are also given. **Mahaaveera** defined operations with zero by saying:

"*ताडितः खेन राषिः खं सोश्षविकारी ह्तो युतः हीनोश्षपि खवधादिः खं योगे खं योज्य रूपकं*"

[GSS Chapter 1 Verse 49**]**

which can be written in symbolic form as:

$a+o = a$, $a-o = a$, $ax0 = 0$ and $a / o = a$.

The last result was incorrect and later it was corrected by great Mathematician Bhaskaracharya (1114 – 1193).

Mahaaveera was the first person to mention that no real square roots of negative numbers can exist. The imaginary numbers were not identified until 1847 by Cauchy in Europe. He was the first Mathematician in the world who made a very significant remark on the square root of a negative number by saying:

"ऋणयोर्धनयोघाते भजने च फलं धनम् ऋणं धनर्णयोस्तु स्यात्स्वर्णयोर्विवरं युतौं
ऋणयोर्धनयोर्योगो यथा संख्यमश्णं धनम् षोध्यं धनमश्णं राषेः ऋणं षोध्यं धनं भवेत्
धनं धनर्णयोर्वर्गो मूले स्वर्णे तयोः क्रमात् ऋणं स्वरूपतोश्ष्वर्गो यतस्तस्मातन्न तत्पदम् ॥"

[GSS Chapter 1 Verses 50-52**]**

Its explanation is "*The squares of a positive as well as negative numbers are always positive and the square roots are positive and negative respectively of these quantities. A negative number is a non square by its nature because there is no real square root of it*". This is the first clear recognition of an imaginary number in Mathematics which had to wait for several centuries for its importance.

Jain culture defines 72 modes of Art (Kala) in which the first is worship of God and the second is Mathematics. This motivated **Mahaaveera** to study Mathematics. **Mahaaveera** was a Jain priest. He gave names to 24 Jain Gurus (Tirthankars) in terms of numbers and some special names like 'Ratna-1, Ratna-2 etc. He counted numbers with a list of 24 decuple terms as:

"एकं तु प्रथम स्थानं द्वितीयं दष संज्ञिकम् तृतीयं षतमित्याहुः चतुर्थं तु सहस्रकं ॥
पंचमं दषसाहस्रं शश्ठं स्याल्लक्षमेव चं सप्तमं दषलक्षं तु अश्टमं कोटिरुच्यतें ॥
नवमं दषकोट्यस्तु दषमं दषकोटयः अर्बुदं रुद्रसंयुक्तं न्यर्बुदं द्वादषं भवेत् ॥
खर्वं त्रयोदषस्थानं महाखर्वं चतुदषं पद्यं पंचदषं चैव महापद्यं तु शोडषं ॥
क्षोणी सप्तदषं चैव महाक्षोणी दषाश्टकं षंखं नवदषंस्थानं महाषंखं तु विंषकं ॥
क्षित्यैकविंषतिस्थानं महाक्षित्याद्विविंषकंत्रिविंषकमथक्षोभं महाक्षोभं चतुर्नयम् अथगणकगुणनिरूपणम् ॥"

[GSS Chapter 1 Verses 63-68**]**

It means "*Ek (1), dasha (10^1), shata (10^2), sahastra (10^3), dasha sahastra (10^4), laksha (10^5), dasha laksha (10^6), koti (10^7), dasha koti (10^8), arbud (10^9), nyarbud (10^{10}), kharva (10^{12}), mahakharva (10^{13}), padma (10^{14}), mahapadma (10^{15}), kshoni (10^{16}), mahakshoni (10^{17}), shankha (10^{18}), mahashankha (10^{19}), kshiti (10^{20}), mahakshiti (10^{21}), Kshobh (10^{22}) and Mahakshobh (10^{23})*".

Chapter 2: परिकर्म व्यवहारः *Basic Arithmetical Operations:* In this Chapter; addition, subtraction, multiplication and division, squares and square roots, cubes and cube roots are defined and elaborated.

This contains some interesting puzzles to explain these concepts. **Mahaaveera** called these puzzles as "*Kanthabharanam*" (ornaments). **Mahaaveera** defined palindromes and their factorization by using multiplication. Some examples are:

11111111	= 152207 x 73	(Kanthabharanam)
1002002001	= 11011011 x 91	
15151	=139 x 109	
12345654321	= 279946681 x 441	
111 111 111	= 12345679 x 9	(Narpalkanthikabharanam)
11000011000011	= 333 333 666 667 x 33	
100010001	= 14287143 x7	(Ratnakanthikabharanam)
1000000001	= 142857143 x 7	(Rajkanthikabharanam)

Throughout the work a place-value system with nine numerals is used or sometimes Sanskrit numeral symbols are used. Of interest in Chapter 1 regarding the development of a place-value number system is **Mahaaveera**'s description of the number 12345654321 which he obtains after a calculation. He had described the number as:

"*Beginning with one which then grows until it reaches six, then decreases in reverse order*".

Notice that this wording makes sense to us using a place-value system but would not make sense in other systems. It is a clear indication that **Mahaaveera** is at home with the place-value number system.

Chapter 3: कालसवर्ण व्यवहारः *Fractions:* This Chapter is related with fractions. This contains rules of transactions, distribution of wealth etc. These rules are used in exchange of gold or mercantile deeds and barter.

Mahaaveera was the the first Mathematician in the world who had given the concept of **Least Common Multiple (LCM)** known as **Niruddha** in those days which is as follows:

"*छेदापर्वतकानां लब्धानां चाहतौ निरुद्धः स्यात् हरह्रत निरुद्धगुणिते हारांषगुणो समो हारः* "
[GSS Chapter 3 Verse 56**]**

Its meaning is "*The **Niruddha** or **LCM** is evaluated by product of common factors of all denominators and their quotients respectively. The new numerators and denominators obtained as products of each original numerator and denominator by the quotient of the Niruddha divided by the denominator give fractions with the same denominator*".

Chapter 4: प्रकीर्णक व्यवहारः *Miscelleneous Problems:* This includes distribution of property amongst the relatives. For this, **Mahaaveera** used '*Ekanshak Bhinna'*, that is fraction whose numerator is 1 representing as a sum of as:

$$1 = (1/2) + (1/3) + (1/3^2) + \ldots \text{ to n terms}$$

and then tending n to "Anant" that is infinity.

Such types of fractions are called Egyptian Fractions and were used in 1127 AD.

Chapter 5: त्रैराषिक व्यवहारः *Rule of Three:*This contains the method of find the fourth quantity if three are known (*Trairashik*). **Mahaaveera** says "if determination, hard work and aim are confirmed, what will be the output ? He answers – success and only success"!

In the same tune, he has given some day-to-day problems of sale and purchase of market goods. For last 1000 years, this rule was in practice in Bharat. In 9th century, after **Mahaaveera**, this rule was borrowed by Arab countries. Al-Beruni included this rule in his text "*Rashikat-al-Hind"* (Wealth from Bharat). Later on, this rule was introduced in Europe by the name "Golden Rule".

Chapter 6: मिश्रक व्यवहारः *Mixed Problems:* This chapter contains rules about ratio of mixing copper with gold for making ornaments, rules of interest on capital. Please see this example:

"If the capital is 10, 20, ..., 100 are invested for the period 1, 2, 3, ..., 10 months respectively and the sum of all interests is 74, find the rate of interest, if it is common for every period? To solve this type of problems, **Mahaaveera** used the following formula of proportion:

$$(a / b) = (c / d) = (e / f) = \ldots = (a + c + e + \ldots) / (b + d + f \ldots)$$

which we learn even today at school level.

Mahaaveera was the world's first Mathematician who gave the concept of **Combination(प्रस्तारयोगभेदस्य सूत्रं)**. The formula for combination is as shown below:

$$^{n}C_{r} = (n!) / [(n-r)!\ (r)!]$$

This formula is given all books of algebra of Class XI of all Boards (State Boards, CBSE, ICSE etc.) of Bharata. The original text is as follows:

"*एकाद्येकोत्तरतः पदमूर्ध्वाधिर्यतः क्रमात्क्रमषः स्थाप्य प्रतिलोमघ्नेन भाजितं सार* "

[GSS Chapter 6 Verse 218**]**

Its meaning is "*The quotient of product of numbers in descending order starting from a number n (total objects) to (n-r+1) divided by product of numbers in ascending order starting from 1 to a number r (choice) is the result*". Such formula came in practice in 1634 AD when it was propounded by French Mathematician and Astronomer Pierre Herigone (1580–1643).

He studied several arithmetic and geometric series also. **Sum of Squares of n terms** of an Arithematic Progression (AP) are computed as shown below:

"*द्विगुणौकोनपदोत्तरकृतिहतिशश्ठांषमुखचयहतयुतिः व्येकपदघ्नामुखकृतिसहिता पदताडितेश्टकृतिचितिका* "

[GSS Chapter 6 Verse 298**]**

It means "*Twice the number of terms (n) in AP is lowered by one and then multiplied by the square of the common difference (d) and is then divided by six. This product is added by the product of the first term (a) and the common difference. This sum is multiplied by the number of terms lowered by one. The sum of this product and square of the first term is calculated. Obtain product of this sum and the number of terms which the required sum of the squares of all (n) terms in given AP*".

In symbolic form, one can write: $\text{Sum} = \left[\left\{\frac{(2n-1)d^2}{6} + ad\right\}(n-1) + a^2\right]n$

Sum of Cubes of n terms of an AP are calculated as given below:

"*चित्यादिहतिर्मुखचयषेशघ्ना प्रचयनिघ्नचितिवर्गे आदौप्रचयादूने वियुता युक्ताधिके तु घनचितिकां* "

[GSS Chapter 6 Slok 303**]**

Its meaning is "*Obtain the product of the sum of simple terms of the series, the first term and the first term subtracted by the common difference. Suppose this is first product. Evaluate the product of square of sum of simple series and the common difference. Suppose this is second product. Thus, sum these products is the required sum of cubes of n terms of an AP is evaluated*".

Chapter 7: क्षेत्रगणित व्यवहारः *Calculations of Areas:* This chapter is devoted to geometry, especially about area. Rules of finding areas of rectangle, square, rhombus and parallelogram are mentioned. **Mahaaveera** has given the formulae for the areas of circle, ellipse, parabola and hyperbola; also about curved surface areas of sphere, cone, cylinder etc. **Mahaaveera** had given a list of 16 primary plane figures which are (i) three varieties of triangles, equilateral, isosceles and scalene; (ii) five varieties of quadrilaterals, equilateral, equidichastic, equibilateral, equitrilateral and inequilateral; (iii) eight varieties of curvilinear figures, a circle, a semi circle, an ellipse, a conchiform area, a concave circle, a convex circle, an outlying annulus and an in lying annulus. **Mahaaveera** had given the rule for finding the volume of frustum like solids. **Mahaaveera** used ð = "10. He borrowed this value from **Brahmasfutsidhanta** of Brahmagupta (598-628 AD). Interestingly, **Mahaaveera** has also mentioned about volumes of various solids using examples.

Mahaaveera had discussed two rules one for approximate and another for accurate results for mensuration. He dealt with various usual plane figures. For ð, he used the Jaina values 3 (gross) and “10 (accurate). **Mahaaveera** was the first Mathematician to deal with an Ellipse which he calls आयतवृत्त (elongated circle) which should be appreciated because for the exact rectification of the ellipse; the Mathematicians had waited for about eight centuries to acquire the powerful tool of calculus. This gives better results than Johannes Kepler's formula (1571-1630). For an ellipse; his accurate results for semi major axis (a) and semi minor axis (b) are:

$$\text{Area} = b\sqrt{(4a^2 + 6b^2)}; \text{ Perimeter} = \sqrt{(16a^2 + 24b^2)}$$

“व्यासकृतिश्शङ्गुणिता द्विसंगुणायामकृतियुता पदं परिधिः व्यासचतुर्भाग गुणष्चायतवृत्तस्य सूक्ष्मफलम् ”
[GSS Chapter 7 Verse 63**]**

When discussing **Rational Scalene Triangle (RST), Mahaaveera** had said that

“जन्यभुजार्धं छित्वा केनापिच्छेदलब्धजं चाभ्याम् कोटियुतिर्भूः कर्णौ भुजौ भुजा लम्बका विशमें”
[GSS Chapter 7 Verse 110.5**]**

Its meaning is that “*Just half of the base of derived rectangle is first divided by the imagined divisor. This quotient and imagined divisor are used to get the next rectangle. The sum of the uprights is the base, two diagonals are the sides and the base of the rectangle is altitude of required **RST***”.

While discussing **Pairs of Rational Isosceles Triangles (PRIT) Mahaaveera** had stated that

“रज्जुकृतिघ्नान्योन्यधनाल्पान्तं शडद्विघ्नमल्पमेकोनम् तच्छेशं द्विगुणाल्पं बीजे तज्जन्ययोर्भुजादयः प्राग्वत् ”
[GSS Chapter 7 Verse 137**]**

It means that “*Obtain the product of the square of the ratio of value of the perimeters and the areas in alteration. Compute the quotient by dividing larger product by smaller one. To get the first pair; this quotient is multiplied firstly by six and then by two and this second product is lowered by one. The first product and lowered smaller product is the first required pair. To evaluate the second pair; the same quotient is multiplied by four and this product is firstly increased by one and then decreased by two which gives the second required pair*”. From these two pairs; the base, equal sides and altitude are calculated for **PRIT**.

The particular case of PRIT is discussed in 17th century by Johann Heinrich Rahn (1622-1676 AD), a Mathematician of Switzerland. However, the general rule is not available till this date.

Mahaaveera had discussed about construction of a **Rational Right-angled Triangle (RRT)** as given below:

“यद्यत्क्षेत्रं जातं बीजैस्संस्थाप्य तस्य कर्णेनं इश्टं कर्णं विभजंल्लाभगुणाः कोटिदोः कर्णाः ”
[GSS Chapter 7 Verse 122.5**]**

Its meaning is “*Each of the different figures that are drawn with the help of a given number namely bijas (seeds) is written down. The known diagonal is divided by the diagonal of the known seeds (elements). It is called quotient. The perpendicular, the base and the diagonal are multiplied by the quotient evaluated earlier which gives the required perpendicular, base and diagonal of a RRT*”. This gives rise of a Circle whose diameter is the hypotenuse of RRTs.

Italian Mathematician Leonardo Fibonacci (1170-1250 AD) was the first western Mathematician who discussed about the construction of a RRT in 1202 AD.

Chapter 8: खात व्यवहारः *Calculations of Excavations:* He gave techniques for calculating areas and volumes. An elegant generalisation of the rule for finding area and volume which clearly reflects the typicalness of the method of averaging was given by **Mahaaveera** is as follows:

"*बाह्यभ्यन्तरसंस्थिततत्तत्क्षेत्रस्थबाहुकोटिभिः स्वप्रतिबाहुसमेता भक्तास्तत्क्षेत्रगणनयान्योन्यम्*
गुणिताष्च वेधगुणिताः कर्मान्तिकसंज्ञगणितम् स्यात् तद्बाह्यभ्यन्तरसंस्थिततत्तत्क्षेत्रे फलं समानीयं
संयोज्य संख्यप्राप्तं क्षेत्राणां वेधगुेणतं चं औण्ड्रफलं तत्फलयोर्विषेशकस्य त्रिभागेनं
संयुक्तं कर्मान्तिकफलमेव हि भवति सूक्ष्मफलम् "

[GSS Chapter 8 Verses 9-11.5**]**

Its translation is "*The values of the base and the other sides of the figure representing the top sectional area are added respectively to the value of the base and corresponding sides of the figure representing the bottom sectional area. The several sums as arrived at are divided by the number of the sectional areas taken into consideration in the problem. The resulting quantities are multiplied with each other as required by the rules bearing upon the finding out of areas when values of the sides are known. The area so arrived at when multiplied by depth, gives rise to the cubical measure designated by the Karmantika result. In the case of those same figures representing the top sectional area and the bottom sectional area, the value of the area of each of these figures is separately arrived at. The values of area thus evaluated are added together and then divided by the number of sectional areas taken into consideration. The question as obtained is multiplied by the depth. This gives rise to the cubical measure designated the Aundra result. If one third of the difference between these two results is added to the Karmantika result, it indeed becomes the accurate value of the cubical contents*".

Its typical example is

"*नवतिरषीतिः सप्ततिरायामष्चोर्ध्वमध्यमूलेशुं विस्तारो द्वात्रिंषत् शोडष दष सप्त वेधोश्यम्* "

[GSS Chapter 8 Verse 16.5**]**

It means "*lengths and breadths of the three rectangular sections are 90, 80, 70 and 32, 16, 10 respectively and depth is 7*". So that

$$K = \frac{(90+80+70)}{3}\frac{(32+16+10)}{3}7 = 10826.67$$

$$N = \frac{(90x80+70x32+16x10)}{3}7 \quad = 11340$$

It should be noted that rectangular sections are not similar.

Chapter 9: छाया व्यवहारः *Calculations of Shadows*: Problems on shadows due to sunlight with some practical cases are given in this Chapter. **Mahaaveera** has used Baudhayana theorem, specially, about the inclination of Sun to horizon. Several formulae and examples are given to find out the length of the shadow in day time from which the time can be evaluated. For instance:

"*समचतुश्रायां दषहस्तघनायां नरच्छायां पुरुशोत्सेधद्विगुणा पूर्वाह्ने प्राक्तटच्छायां*
तस्मिन्काले पष्चात्तटाश्रिता का भवेद्गणकं आरूढच्छायाया आनयनं चेत्कथयं "

[GSS Chapter 9 Verse 38.5, 39.5**]**

Mahaaveera gave special rules for the use of permutations and combinations which was a topic of special interest in Jain Mathematics. He also described a process for calculating the volume of a sphere and one for calculating the cube root of a number. He looked at some geometrical results including right-angled triangles with rational sides. Among topics **Mahaaveera** discussed in his treatise were operations with fractions including methods to decompose integers and fractions into unit fractions. For example: 2/17 = 1/12 + 1/51 + 1/68. He examined methods of squaring numbers which, although a special case of multiplying two numbers, can be computed using special methods. He also discussed integer solutions of first degree indeterminate equation by a method called Kuttaka. The Kuttaka (Cyclic or Pulveriser) method is based on the use of the Euclidean algorithm but the method of solution also resembles the continued fraction process of Euler given in 1764. The work Kuttaka, which occurs in many of the treatises of Bharatiya Mathematicians of the classical period, has taken on the more general meaning of Algebra. **Mahaaveera** also attempts to solve certain mathematical problems which had not been studied by other Bharatiya mathematicians. For example, he gave an approximate formula for the area and the perimeter of an ellipse. In Hayashi writes: *The formulas for a conch-like figure have so far been found only in the works of **Mahaaveera** and Narayana.* It is reasonable to ask what a "conch-like figure" is. It is two unequal semicircles (with diameters *AB* and *BC*) stuck together along their diameters. Although it might be reasonable to suppose that the perimeter might be obtained by considering the semicircles, Hayashi claims that the formulae obtained: were most probably obtained not from the two semicircles AB and BC.

He discussed techniques for solving linear, quadratic as well higher order equations. **Mahaaveera** solved higher order equations of *n* degree of the forms:

$$ax^n = q \quad \text{and} \quad a\frac{x^n - 1}{x - 1} = p$$

Mahaaveera expressed characteristics of a cyclic quadrilateral. He also established equations for the sides and diagonal of Cyclic Quadrilateral. If sides of Cyclic Quadrilateral are *a, b, c, d* and its diagonals are *x* and *y* while

$$x = \sqrt{\frac{ad + bc}{ab + cd}(ac + bd)} \quad \text{and} \quad y = \sqrt{\frac{ab + cd}{ad + bc}(ac + bd)}$$

Then, $xy = ac + bd$

Conclusion: Doubtlessly; **Mahaaveera** occupies a unique place in history of Mathematics in Bharat and abroad. His contributions towards imaginary numbers, lowest common multiple, combinations, solution of algebraic equations and their applications in practical life of humanity, in determining the areas of various strange and unfamiliar figures are of great importance. Whoever goes through the **GSS**; definitely becomes interested in ancient literature of mathematics. By **GSS,** Mathematics acquired its own identity. It is said that before **Mahaaveera**, Mathematics was in the garb of Jyotish due to religious rituals. **Mahaaveera** gave the subject a form, an independent identity and existence. He also emphasized theoretical and practical implications. **Mahaaveera** established a prominent centre for learning in South Bharat Karnataka. One may say that he earned an esteemed place in the galaxy of Bharatiya Mathematics. He had written few more books which are rarely available.

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Govindaswami

A virtuous Indian Mathematical Astronomer

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Introduction

The earliest expression of mathematical understanding is inextricably linked with the origin of Hinduism. Mathematics formed an important part of the Vedas – the original Hindu scriptures - associated with spiritual activities and the study of "Ganit " or mathematics was given special importance in the Vedic Period. "Vedang Jyotish " says , " Just as the feathers of peacock and the jewel – stone of a snake are placed at the highest point of their body so is the position of mathematics the highest amongst all branches of Vedas and Shastras. "

Hindu mathematicians, from Vedic times to the modern age have been in the forefront of making seminal contribution in the field of mathematics. Here's some of the most celebrated names in the history of Mathematics – *Aryabhatta, Bhaskara, Haridatta, Govindaswami, sankarnarayana, Parmesvara, Damodara, Govinda brahmachari, Nilkantha, Sankara* and many others.

Govindasvami, (800 - 860 A.D.) a great mathematical astronomer of the ancient India. He was born in India in about 800 A. D. and after living a glorious life of about 60 years he died in India in about 860 A. D. He is also famous as Bhatt Govinda as well as Govindaswamin . He worked a lot in the world of Mathematics and Astronomy. It is unfortune that not enough is known about his life. His most famous treatise was Vyakhya , a Bhasya on Mahabhaskariya.

Mahabhaskariya , a creation of Bhaskara 1 is an eight chapter work on Indian mathematical astronomy and includes topics which were fairly standard for such works at that era. The discussion in this book contains the topics such as the longitudes of the planets, conjunctions of the planets with each other and with bright stars, eclipses of the sun and the moon, rising and setting and the luner crescent.

Govindaswami wrote The Bhashya in about 830 at his very young age of about 30 , which was a commentary on the Mahabhaskariya. It contains many examples of using a place – value Sanskrit system of numerals. One of the most interesting aspect of this commentary, is Govindaswamin' s construction of sine Tables. These tables are based on sexagesimal sectional parts of the twenty - four tabular sign differences from the Aryabhatt. These lead to more correct sine values at the intervals of $90^0 / 24 = 3^0\ 45'$. In the commentary Govindaswami found certain other empirical rules relating to computation of sine differences in the argumental range of 60 to 90 degrees.

Theory of Pulveriser used by Govindaswami

The indeterminate equations of the type

$(ax \pm c) \div b = y \qquad (i)$

or $N = ax + b = cx + d \qquad (ii)$ is described as pulverisers. (kuttakara)

Pulverisers are divided into two different kinds . Pulverisers like (I) are considered as non – residual and those like (ii) are known as residual pulverisers.

Following illustrations will give the idea when pulverisers arise for solution,

Illustration (I) : **6 is multiplied by an unknown number and the product is increased by 8 and then the obtained sum is divided by 11. If the division is exact what is the unknown multiplier and what is the resulting number** ?

This will raise the non- residual pulveriser having a = 6, c = 8 and b = 11. This can be quoted as ab + c = x, where a is called "dividend " , b is called " quotient " and c is called the " interpolator " (remainder or residue). If interpolator is negative , it is technically called " *gata* " (shortage) and if it is positive , it is called " *gantavya* " (surplice or excess) .

Here taking unknown number as x , pulveriser (6 x + 8) – 11 = y is obtained for getting solution. Obviously x and y have least integral values.

Illustration : (2) What is the smallest number which yields 7 as remainder when divided by 12 and 6 when divided by n 17 ?

This raises the residual pulveriser. Obviously here pulveriser will be

$$X = 12a + 7 = 17b + 6.$$

Preliminary operation :

"Divide out the dividend (multiplier) and diviser by the non – zero remainder of their mutual division. The resulting dividend and divisor are then said to be mutually prime to each other."

"When the interpolator is negative or positive interpolator is exactly divisible by the non- zero remainder of the mutual division, it should be considered that the given interpolator corresponds to the true non – abraded values of the dividend and divisor and it will be possible to proceed with the actual non – abraded values of the dividend and divisor to solve the pulveriser. Here the given interpolator corresponds to the abraded values of the dividend and divisor and hence we must proceed with their abraded values. "

" Let a = kA and b = kB , where k is the Highest Common Factor of a and b .
If now c = kC , then as per the above rule we need to solve the pulveriser

$$(kAx \pm kC) \div kB = y$$

or $$(Ax \pm C) \div C = y ,$$

and if c is not divisible by k , then

$$(Ax \pm c) \div B = y .$$

In general the puveriser is said to be wrong when the interpolator is not divisible by the H. C. F. of the dividend and the divisor. In the present case in a particular astronomical problem in which the dividend represents the number of revolution of a planet, the divisor represents the number of the civil days and interpolator represents the residue of the revolution of the planet. Hence in such an astronomical problem the residue of the revolution depends upon whether it has been obtained by using the actual values of the revolution number and the civil days or by using their abraded values. Hence above rule can be considered to be justified.

The method of solving the pulveriser :

(A) First set down the dividend and then set down the divisor below it. Then perform their mutual division. Write down the quotients of mutual division one below the other - second one under the first , the third under the second and so on. Carry on the mutual division till the reduced divisors are different from zero. If the number of quotients thus noted are even then obtain the number , *mati* (residue) using the following rule and if it is odd then obtain the *mati* contrarily

Rule : When the interpolator is negative , divide the intrapolator by the reduced dividend, then subtract the resulting remainder from the reduced dividend, then multiply the remainder obtained by the reduced divisor, then increase the resulting product by the interpolator and then divide the resulting sum by the reduced dividend, the quotient is the *mati.*

When the interpolator is positive, diminish the reduced divisor by 1 (one), and then multiply it by the interpolator , divide the product by the reduced dividend and then divide the quotient by the reduced divisor, the remainder is the mati. In case the remainder is zero then the divisor itself is the mati.

Multiply the reduced dividend by the *mati*, then subtract the negative interpolator (or add the positive interpolator) to that product and then divide that difference (or sum) by the reduced divisor. Write down the *mati* under the chain of the quotients and also write down the quotient below it.

Multiply the upper number by the last but one number of the chain and add the last (lowermost) number of the chain to this product and then discard the last number. Repeat this process again and again until there are left only two numbers in the chain.

Among these two numbers divide the upper number by the divisor and lower number by the dividend (if it is possible). The remainders thus obtained respectively denote the days etc. , and the revolutions etc. , which are the required quantities .

Let us try to understand the rule clearly by following illustration (3).

Illustration (3) : The residue of the revolution of Saturn is 24 ; find the days and revolutions performed by Saturn , given that the formula for the Sun's revolutions for A days is 36641 A / 394479375.

Let x be the unknown days and y be the unknown revolutions performed by Saturn in x days. Then we need to solve the following pulveriser :

$$(36641 x - 24) \div 394479375 = y$$

We can check and decide that 36641 and 394479375 are mutually prime , so we proceed with these numbers

Dividend		divisor		quotient		
394479375	=	36641	×	**10766**	+	2369
36641	=	2369	×	**15**	+	1106
2369	=	1106		**2**	+	157
1106	=	157	×	**7**	+	7
157	=	7	×	**22**	+	3
7	=	3	×	**2**	+	1

From above calculations it will be clear that mutually dividing 36641 and 394479375 until the remainder is 1 (means non – zero) and writing down the successive quotients one below the other, we get -

10766
15
2
7
22
2

The reduced dividend and and reduced divisor are 1 and 3 respectively. Now as number of the quotients listed one below other are even (6 - six) and interpolator is negative (– 24) , we follow the rule for the negative interpolator and thus obtain *mati* = [3 –{ (– 24) ÷ 1 }] ÷ 1 = 27 . Multiplying 1 by 27 and subtracting 24 from the product, we get 3 which is divided by the reduced divisor 3 yields 1 as the quotient.

Writing down the mati and this quotient under the chain of quotients, We get

10766
15
2
7
22
2
27
1

Reducing the chain we successively obtain 2 × 27 + 1 = 55 , 55 ×22 + 27 = 1237, 1237 × 7 + 55 = 8714 and so on.

The chain reduces as following

10766	10766	10766	10766	10766	3108044439 (multiplier)
15	15	15	15	288689	288689 (quotient)
2	2	2	18665		
7	7	8714	8714		
22	1237	1237			
55	55				
27					

Dividing 3108044439 by 288689 the remainder is 346688814 and dividing 394479375 by 36641 the remainder is 32202.

Hence $x = 346688814$ and $y = 32202$.
These are the least integral values satisfying the equation.

Therefore the unknown days for the Saturn are 346688814 and the revolutions performed by Saturn are 32202.

(B) When the residue of the revolution is given in terms of signs, degrees, minutes etc., then multiply those signs etc., by the divisor and divide the product by the number of signs etc. , in a revolution and the quotient obtained is the residue of the revolution**.**

Suppose for example that the residue of the revolution of the sun is given to be 4 signs, 28 degrees and 20 minutes . Now 1 sign is 30 degrees. So in this case we can calculate that total minutes in the revolution are $60 [30 \times 4 + 28] + 20 = 8900$. So we multiply 8900 by 210389 (the divisor in this case) and divide by 21000 (the number of minutes in a revolution) because the formula for the Sun's revolutions corresponding to A days is 576 A / 210389 . In this way we get 86688 as the quotient which the radius of the revolution.

To find the days and the revolutions performed by the Sun in this case we now need to solve the pulveriser

$$(576 x - 86688) \div 210389 = y.$$

Following the above method we can obtain the solution easily.

(C) The dividend should always be of the denomination as the interpolator which has been added or subtracted. So when the interpolator is the residue of the sign, etc then the dividend should be multiplied by the number of sign, etc. in a revolution. When the dividend as thus is multiplied by the number of sign, etc , is not prime to the divisor then they should be made prime to each other by dividing them by the non - zero remainder of the mutual division of the first two.

(D) When the dividend is greater than the divisor, divide the dividend by the divisor and set down the obtained quotient in a separate space. Then treating the remainder of the division as the new dividend find the solution by above given method and solution of the original will be apparently obtained. Let us understand this with the following illustration.

Illustration (4) : Solve the pulveriser (30x –- 1) ÷ 7 = y.

Since the dividend 23 is greater than the divisor 7 , we divide 30 by 7. Thus we get quotient 4 and and remainder 2, Treating 2 as the new dividend we solve the pulveriser $(2x - 1) \div 7 = 3$..

The chain of the quotient is 3 and the reduced chain is 4
1 (mati) 1
1

Adding the lower number 1 to the product of the upper number 4 and the quotient 4 obtained in the bigining, we get 4
17.

Hence the solution is $x = 4$ and $y = 17$.

(E) When the residue of the sign, etc. is known , assuming the number of signs, etc. , in a revolution as the dividend and applying the process of the pulveriser first the residue of the revolution can be found out and then applying the same process the revolutions of the planet can be obtained from the residue of the revolution.

Illustration (5) : The residue of the sign of the Sun is 154168. Obtain the days and the revolutions and signs of the Sun's longitude.

Let x = residue of the revolution of the Sun and y = signs of the Sun's longitude. Hence raises the pulverise (12 x – 154168) ÷ 210389 = y.

Solving by above method we get x = 82977 and y = 4

Now new pulveriser (576 m – 82977) ÷ 210339 = n will take place , where M = days required and n = the sun's longitude.

Solving it by above method m = 17564 and n = 5800.

Some examples referring to Gowindaswami's commentary.

Illustration (6) Given that 100 minutes of the eight sign are to be traversed by the Sunsay quickly, after carefully considering, O intelentone, if the Ganit of Asmaka is known to you, all the years that have elapsed this day since the beginning Kaliyuga . Also say the number of the daysthat have elapsed since the bigining of the Kaliyuga.

Solution : Here according to Bhaskara 1 's interpretation, the part of the revolution to be traversed by the Sun = 7 signs 100' . The corresponding residue of the revolutions = 123707. This is positive.

We have therefore to solve the pulveriser

(576 x + 123370) ÷ 210389 = y ,

Where x = the required *ahargana* and y – 1 = the number of years elapsed.

Mutually dividing 576 and 210389 and taking 1 for the optional number *(mati)*

We get the chain as

365
3
1
6
2
4
1 *(mati)*
61851

Which reduces to 1310408037
3587617

Dividing 1310408037 by 210389 and 3587617 by 576 , we obtain remainders 105345 and 289. Therefore x = 105345 and y = 289

Hence the required *ahargana = 105345 and number of the years elapsed = 288*

Note : According to Govindaswami's interpretationthe part of the revolution to be traversed by the Sun = 4 signs 1 ^{0} 40' and The corresponding residue of the revolution = 71104 .

The resulting pulveriser is

(576 x + 71104) ÷ 210389 = y ,

Of which the solution is x = 186889 and y = 512 .

Hence the required *ahargana = 186889 and number of the years elapsed = 511.*

Illustration (7) : The sum of the mean longitudes of Mars and moon is calculated to be 3signs, 7 degrees, 9 seconds and 6 thirds. O you, well verses (well wisher) quickly say the ahargana (days)

Solution :

The revolution number of Moon = 57753336 .
The revolution number of Mars = 2296824 .
Sum = 60050160 .
The number of civil days in a yuga = 1577917500 .

H. C. F . of 60050160 and 1577917500 is 60 . Therefore , the abraded sum of the revolution numbers of Moon and Mars = 60050160 ÷ 60 = 100836 .

The sum of the mean longitudes of Moon and Mars = 5 signs 7^{0} 9 ' 9 " 5 "'
= 33944946 thirds

Now in case of the longitude of a planet is given in terms of signs etc , the signs, etc are multiplied by the abraded number of civil days in a *yuga* and the product is divided by the number of signs, etc. (in a circle) and the quotient is stated to be the residue of the revolutions.

Therefore, the residue of the revolutions = 11480265 .

Hence now we have to solve the pulveriser

$$(1000836 x - 11480265) \div 26298625 = y ,$$

Where x = required ahargana (days)

Solution of this pulverisert is x = 10157490 ,

y = 386459 .

The require ahargana = 10157490.

Note : If the mean longitudes of Moon and Mars are needed , then can be easily calculated from this ahargana

Conclusion

We here had tried to discuss Astronimical side of Govindaswami . Still much of his contribution remains to be fully understood and made intelligible to the modern students. His original work on Astronomy and Mathematics which is referred to as *Govindakriti* , by later is yet to be recovered. He has enunciated a set of different formulae being laid down for different step – lengths in the modern notation, in terms of the finite difference operator Δ. Govindaswami's formula takes the form - $f(x + nh) = f(x) + n \Delta f(x) + \frac{1}{2} n (n-1) [\Delta f(x) - \Delta f(x-h)]$, which is a particular case (upto the second order) of the general Newton – Guass interpolation formula.

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Shripati

The creator of "Ganittilaka"

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Introduction

Shripati (1019 – 1066) was the most prominent Indian Astronomer and Mathematician, who is famous for his contribution in mathematics by his creation of " *GANITTALIKA* ", an arithmetical treatise containing the rules of signs for addition, subtraction, multiplication, division, square, square root, cube, cube root of positive and negative quintiles. His other works are (1) *Dhikotidakarana*, a work on solar and lunar eclipses ,(2) *Dhruvamanasa*, a work on finding planetary longitudes, eclipses and planatery transit (3) *Jyautisratnamala*, a book based on *Jyotisaratnakosha* of *Lalla*, (4) *Daivajna - vallabha*, a work on astrology which some historian claim to be written by Varahmihira But nobody yet had come along the proof to prove the fact, (5) Jatakpaddhati, a book on the computation of the strength of the planetsand astrological places in astrology, (6) *Siddhantsekhara*, a book on astronomy containing the work on Arithmatic, Algebra and sphere. His work on solving quadratic equations is worth to take into consideration. He had given a well known identity :

$$\sqrt{(x + \sqrt{y})} = \sqrt{[x + \sqrt{(x^2 - y)}] / 2} + \sqrt{[x - (x^2 - y)] / 2} .$$

Life of Shripati

Exact date about his birth is not known, but from his different creations we can guess that his life interval may be (1019 – 1066) A. D . as he constructed *Dhruvamanasa* in S'aka 978 . There he states that his father is *Nagadeva*, the son of *Bhatt Kesava.* From *Siddhantsekhara*, it can be decided that he was a *Brahmin*. From *Ganittilaka* it can be decided that he was Shaiva by religion. From his book Jatakpaddhati it can be decided that he was a teacher (*Guruji*) having some students under him and on the request of a pupil he put up his creation. From his astrology work it can be said that he wrote them in the place, *Rohinikhanda* which may be perhaps his native place. From his astronomical works it can be said that he was not only a celebrated astronomer but he was an astrologer also.. It can be observed that he lived in between the time of *Majula,* the author of *Laghumanasa* (932 A. D.) and *Bhaskaracharya,* the author of *Siddhantsekhara* (1040 A. D.)

Ganittilaka :

We shall try to discuss some important aspects of Ganittilaka , a well known mathematical work of Shripati , which contains laws of Arithmetic (Patiganita) and a nice work on solving Arithmatical problems . It is entirely written in Sanskrit verses. we shall discuss the ideas and methods represented by Shripati. He has tried to explain the laws with Sanskrit verses and each verse is followed by the explanatory commentary in Sanskrit with the help of the illustrations.

Starting by salutation to the nature of the soul, Shripati indicates the subject and the purpose of composing the work on Pati – Ganita.

Big numbers

He represents the number series with the **f**irst number 1 and states that by " *sunya vruddhya vardhamanatam* " means adjusting zeros on right of it the big numbers arise. Also he gives the names

of the numbers of the increasing series, eka (1), Dash (10), Sata (10^2) , Sahasra (10^3) , Ayut (10^4) ,Laksh (10^5), Prayuta (10^6) , koti (10^7) , Arbuda (10^8) , Padma (10^9) , Kharva (10^{10}) , Nikharva (10^{11}) , Mahasroja (10^{12}) , Sanku (10^{13}) , Antya (10^{14}) , Madhya (10^{15}) . and Parardha (10^{15}). Here the number super scribed shows the number of zeros placed on the right side of the 1 eg. 10^8 = eight zeros on right side of 1 = 100000000. Also he relates the bigger numbers with smaller numbers, such as “” Ekostau sunyani Arbudam,” - dash Kotihi means 1 with eight zeros is Ayutam which is ten times Koti and shows that each zero placed on right of the number increases it by ten times.

It seems that as described in different verse that numbers Ek (one) , dvaya (two) , tri (three) , Chatur(four), Panch (five), Shad (six) , Sapta (seven), Ashta (eight) and nava (nine) are familiar and they are in an order and this order is used for addition in the next session. Here it is clearly followed that zero is used for place value. Also in the discussion of Sankalitam (addition) it is clearly explained that when zero is added in any number or any number is added in zero number remains unaffected I,e, for any number m, $m + 0 = 0 + m = m$ and $m - 0 = m$.

Sankalitam - Vyakalitam (addition - subtraction)

Now he explains the process of addition –Sankalitam.. Using order addition is carried out in this session. Taking an illustration it is shown - “.Sapta madhye Kshipta ashtau jataha panchadash “ means 7 added to 8 becomes 15 . Here it is noted “ kramotkramabhyam meelita “ means by passing through the order 8 added to 7 makes 15. Also it is shown that to add more numbers first their unit digits must be added and then addition is to be finished. Here two methods “karma “ and “ Utkrama “ are explained and then use of the order and the addition - (samuchchaya) how subtraction will be carried out is explained in a verse and taking an illustration method of subtraction is clerified. Onwards he explains to add and subtract the fractions with common denominator and different denominators also.

Gunakar (Multiplication)

Multiplication by single digit is explained in detail and then how carry is to be added on left is explained. Shripati had given four methods of multiplication (I) *Kapatsandhi* : This is the method of carrying out the product by each digit of the number by which it is to be multiplied (*kapat* - part).It is worth noting that first multiplication by ten’s digit is carried out. Then multiplication by unit digit if started and it is sifted one place on right side and then addition (*sandhi*) is carried out .
(2) *Tatasth* (steady) In this method multiplier is kept steady and cross multiplication is applied. (3 – 4) *sthanvibhaga* and *rupavibhaga are the methods using algebraic methods.*

Bhagahar (Division)

Explanation of division method is very elementary . But one thing is worth noting that Sripati says , “ Remove the common factors , if any , from hara (divisor) and bhajya (dividend) and then divide the pratiloma (inverse) order. “ Going further detailed process is well explained. He also explains the division by a fraction as “”Adhaharena urdhvashan, urdhvaharenadharam haram hanyat “ means to divide by a/b, multiply by b/a. Also he discusses *Bhaga-bhaga* , which contains division in succession.

Varga *(* Square *)*

Shripati gives three methods for finding the square of a number . These are in detail and interesting also.

Method (1) :

In the first method he states ,

" *After the last digit is squared, double this last should be multiplied by the rest of the digits respectively."*

This method is the another form of the modern algebraic method in another form. In modern method of algebra, $(a + b)^2 = a^2 + 2 \times a\ b + b^2$. But if we transfer it into the form as $(a + b)^2 = a^2 + b^2 + 2 \times a \times b$, it will turn into Shripati's method but in another form. Hence let us understand with his illustration.

Illustration : Find the square of 163 .

Square of 1 is 1. Double of 1 is 2 which multiplied to 6 and 3 gives 12 and 6 respctively , which gives 126. Now square of 6 is 36 and its double is 12 which yields 36 when it is multiplied by 3. When this joins with square of 1 gives 136. Lastly square of 3 is 9.

Shripati represents his eleven steps as following :

(I)	(ii)	(iii)	(iv)	(v)	(vi)
163	163	163	63	63	63
1	12	126	126	126	126
1	1	1	1	1	136

(vii)	(viii)	(ix)	(x)	(xi)
126 $_3$ 6	123 $_3$ 6	123 $_3$ 6	126 $_3$69	126 $_3$69
136	136	136	136	136
				26569

Any one can easily understand that the steps (vi) and (ix) of Shripati are

uselessly repeated.. Many observers had criticized that Shripati had followed wrong way to put up this method, because in his third method he explains the definition of square and he uses it in this method. It is a wrong logic. Though the argument seems to be true, Shripati may have his own idea or reason for this.

Method (2)

This method is also very much interesting. Shripati describes the method as under,

" The product of the difference and the sum of the number to be squared and the assumed number, when combined with the square of the assumed, gives the square required . "

Let us understand the method with an illustration.

Illustration : Find the square of 56 .

As per this method in this case assumed number will be 4 because when it is added to 56, it turns into 60.

So square of 56 $= (56 + 4) \times (56 - 4) +$ sq. of 4 is to be calculated .

It gives Square of 56 $= 60 \times 52 + 16$

$= 3120 + 36$

$= 3136$.

Note : Here in this case assumed number can be selected to be 6 also , as 6 is subtracted from 56, it turns to be 50.

So square of 56 $= (56 + 6) \times (56 - 6) +$ sq. of 6

$= 62 \times 50 + 36$

$= 3100 + 36$

$= 3136$.

Illustration : Find the square of 984.

In this case assumed number will be 16 because when it is added to 984 , it will turn it into 1000.

So square of 984 $= (984 + 16) \times (984 - 16) +$ sq. of 16 .

It gives Square of 984 $= 1000 \times 968 + 256 = 968256$

Note : In modern algebra $x^2 = (x + a)(x - a) + a^2$, where x is the given number and a is the assumed number. Obviously the assumed number is so selected that its sum or difference with given number is a number containing zeros at the end and the square of the assumed number is easily available.

It is worth to take note of the fact that this method seems reflected in " *Vedic Mathematics* " of Shri Shankaracharya Krishnatirthaji Maharaj as a method known as " *Yavadunam – Yanadadhikam* " with " *Aanurupyena* ". In that method either (x + a) or (x – a) works as the base, which contain zeros at the end. If (x – a) contains zeros on its right side, it is taken as base, then (x + a) is the increased quantity (*Adhikikritya*) and if (x + a) contains zeros, it is considered as the base and (x – a) is the decreased quantity (Unikritya) and then multiplication by the base removing zeros at the end is " proportionately " (*Aanurupyena)* and then addition of a^2 is " *vargam cha yojayet,* " the adjustment of the square on right side.

See his work of above illustrations-

$56^2 = 5(56 + 6) / 6^2 = 310 /{}_3 6 = 3136$ or

$56^2 = 6(56 - 4) / 4^2 = 312 /{}_1 6 = 3136$.

$984^2 = (984 - 16) / 16^2 = 968 / 256 = 968256$

Method 3

In the third method Shripati defines the square of a number.

He denotes -

" to get the square of a number *multiply the given number by itself*. "

Hence square of 12 is (12 × 12) = 144 .

Means now in this method Shripati defines what is a square of the number : Square of a number m is m × m.

This raised a chance for the critics to put up a charge against Shripati that his logic is not understandable, why he defines the square in the last method, in spite of putting it as the first method.

Ghana (Cubing)

Shripati gives four methods for finding the cube of a number. Among these three first one is worth drawing our interest. It relates to the modern algebraic method which is now a day's well known, but it is in another format. Hence it is discussed below.

Method 1 : " Find the cube of the last (digit) , then the product the square of this last (digit) multiplied by three and the succeeding (digit), then the product of square of this succeeding (digit) multiplied by last (digit) and three and then the cube of the succeeding (digit). Now add them after each is placed one place before the other, gives the cube of the number "

Illustration : Find the cube of 317:

For finding cube of 31 above four steps are as following -

(a) cube of 3 = 27 , (b) square of 3 x 3 x 1 = 27 ,

(c) (square of 1 x 3) x 3 = 9 and (d) cube of 1 = 1.

Placing one step before

27
 27
  9
   1
______
$27\ {}_{2}7\ 9\,1$

Hence cube of 31 = $27\ {}_{2}7\ 9\,1$

For fonding the cube of 317 above narrated four steps are -

(a) cube of 31 = $27\ {}_{2}7\ 9\,1$ (b) Square of 31 x 3 x 7 = 20181

(c) (square of 7 x 3) x 31 = 4557 and (d) cube of 7 = 343

Placing one step before

```
2 7 7 9 1
  2
  2 0 1 8 1
    4 5 5 7
       3 4 3
----------------
3 1 8 5 5 0 1 3
```

Thus by this method cube of 317 = 31855013.

This is same as $(m + n)^3 = m^3 + 3m^2 n + 3mn^2 + n^3$

Now taking m = 10 a and n = b , we get

$(10a + b)^3 = 10^3 \times a^3 + 3 \times 10^2 \times a^2 \times b + 3 \times 10 \times a \times b^2 + b^3$

$(10a + b)^3 = a^3 / (a^2 \times 3) \times b / (a \times b^2) / b^3$

This seems so simple today, was not so simple in that era, but it is very much important that he can draw out such rule at that time when algebra was not in use. It is also clear that slashes evidently explain the reason of placing one place before. It is due to multiplication of the powers of 10 in succession.

Method 2

This method as Shripati describes is to find the cube of a term of an A. P. whose first term is 1 and whose common difference is also 1. In this method he statesthat,

" The cube is obtained by adding together (I) The last term multiplied by 3 as well as by the preceding one (*mukh*), and (ii) the cube of this *mukha*, with 1 added to it."

Obviously this method relates to the set of natural numbers. Here " mukha " is the previous number to the given number, whose cube is to be obtained. Algebraically we can represent it as $a^3 = 3 \times (a - 1) \times a + (a - 1)^3 + 1$. This method is much more useful if you know the cube of its preceding number (*mukha*). We know that cube of 30 is 27000. Now 30 is *mukha* for 31, so it is easy to find out the cube of 31.

Cube of 31 $= 3 \times 30 \times 31 + 27000 + 1$

$= 2790 + 27001 = 29791$.

Same way cube of 32 $= 3 \times 31 \times 32 + 29791 + 1$

$= 2976 + 29792 = 32768$

and so on we can get the cubes of all remaining inegers greater than 30..

A small change here will be helpful to raise the interest of people interested in mathematics. If we consider " a " as *mukha* , then a is *mukha* for its successive number $(a+1)$. Now if cube of a is known, the formula for finding the cube of

$(a + 1)$ is – $(a + 1)^3 = 3 \times a \ (a + 1) + a^3 + 1$.

Method 3

Method defines the cube of a number. It states that to get the cube of a number " *multiply the given number by itself and then multiply the result by the given number.* "

Thus in this method Shripati defines the definition of the cube of a number a number. As per this method cube of m is $m \times m \ m$. Thus after using this definition now in this method he defines the cube of a number, which becomes a reason for the critics to raise the question against the non - logical order of Shripati.

Method 4

This method states , " Thrice the rashi (given number) multiplied by its two parts, when united with the cubes of each of those parts give the required cube of given rashi . "

Cube of 1 and 2 are 1 and 8 respectively and 1 and 2 are the two part of 3 (rashi). So cube of 3 = $3 \times 1 \times 2 \times 3$ + (cube of 1) + (cube of 2) = 18 + 1 + 8 = 27. Now 3 and 10 are two parts of new rashi 13 and their cubes are known. Hence cube of 13 = $3 \times 3 \times 10 \times 13$ + (cube of 3) + (cube of 10) = 1170 + 27 + 100 0 = 2197 and so on we can find the cube of any number we need.

Fractions :

Shripati has used the words *amsa, amsaka* and *lava* for numerator and he had used the words *cheda, chhedaka, chhid* and *hara* for denominator .He had used cardinal numbers as well as ordinal numbers to denote the unit fractions . For example 1/3 is tyamnsa, 1/6 is sadamsaka or sat- cheda and 1/3 is trilava etc. as well as 1 /7 is saptambhaga . Etc. He had dealt with compound fractions $a \pm (b/c)$ as well as $(a/b) \pm (c/d)$ also . He also deals with the problems like $a \div (b/c)$ and $(a/b) \div (c/d)$. In the treatment of kalasavarana, the reduction of the fractions his one action is worth to draw attention that He had wrongly defined the division by zero, . otherwise his dealing with simplification of fraction is very much effective. He defines, (1) $m + 0 = m$, (2) $m - 0 = m$, (3) $m \div 0 = 0$, (4) $m \times 0 = 0$, (5) $m + 0 = 0$ (6) $0 \div 0 = 0$ (7) $(0)^2 = 0$ and , (8) $(0)^3 = 0$. Also he defines that the square root and cube root of zero are zero in both the cases.

Proportion :

Under this topic Shripati has dealt with trirashika (the rule of three) and panch rasika (the rule of five) . He had noted two types of the varieties of trairasika (1) sama (direct proportion) and (2) vyasta (invers proportion).Onwards he had treated the problems on rule of seven, nine, eleven *sashika* and further more and he had dealt with various problems on finding interest.

Tail of the book :

At the end of the book it contains some tables of different Vyavahara (dealings) . These tables represent the relations between the units of different fields.

e.g. ,Trading of gold, measurement of length, measurement of area, measuremenr of different weight etc.

Conclusion :

In Ganittilaka Shripati had dealt with Parikramastakam I,e, eight basic processes - Addition, subtraction, multiplication, division, square, square root, cube and cube root of whole numbers. And in addition he had included fractions and proportions. It can be observed a great task of Shripati providing lots of illustrations for explanation. Not only that but we can find many varieties of examples in the book. He did not use only one type of spicies but he had justified animals as well as birds also.

It is not easy to decide from which source he would have collected such different varieties of examples. It is very difficult to frame such type of examples in a short time. Lots of time needed to frame such examples. It relacts a high potentiasl of Shripati. The whole book contains continuous verses and they are not divided into part means chapters, but each verse follows with commentary in Sanskrit accompanied by illustration which helps the individual in understanding it .

A commentary on Ganittilaka was written and published by Simhatilaksuri .

HE observes that Shripati had closely followed Trisati , a creation of Shridhara and also he had borrowed many things from it. Even some examples are borrowed from it as it is and many of the examples are modified up to some extent .

Lastly it can will be worth to salute the great work done by Shripati.

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Āryabhaṭa II - A Brief Note

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1. Introduction

In the development of mathematics and astronomy in India, the medieval mathematicians and astronomers who contributed very significantly are Āryabhaṭa I (b.476 CE), Brahmagupta (628 CE), Bhāskara I (629 CE), Mañjula (932 CE), Śrīpati (c.10th cent), Āryabhaṭa II (c.10th cent), Bhāskara II (b.1114 CE), Gaṇeśa Daivajña (1520 CE) and the later medieval remarkable stalwarts of the Kerala school.

The role of Āryabhaṭa II is generally underplayed. All the same, certain features in his text, *Mahāsiddhānta* are unique and significant.

2. Date and Time

Āryabhaṭa II does not mention his time or place unlike Āryabhaṭa I who atleast cites his date and that his system of astronomy was honoured at Kusumapura (identified with Patna in Bihar).

However, we can estimate a time-interval in which Āryabhaṭa II lived. S.R.Sarma (1966) determines lower and upper bounds for this period. Bhāskara II refers to our medieval astronomer in the following passage of *Siddhānta Śiromaṇi* .

Āryabhaṭādibhiḥ sūkṣmatvārthaṃ dṛkkāṇodayāḥ paṭhitāḥ |

– *Si.Śir.Spṣṭā.Śl*.65

Āryabhaṭa I in his *Āryabhaṭīyam* has given the durations of the rising of only *rāśis* and not of *drekkāṇas* (one third of a *rāśi*). Āryabhaṭa II in his *Mahāsiddhānta* (Ch.IV, 38-41) has given the *udayamānas* of the *drekkāṇas* . This shows that Bhāskara II indeed refers only to Āryabhaṭa II and hence the latter lived before the 12th century, the lower limit.

To estimate the upper limit of Āryabhaṭa II's period, S.R.Sarma mentions the internal evidence that *Mahāsiddhānta* (*MS*) itself refers to Parāśara , Garga and "*Vṛddha*" Āryabhaṭa . Among these, Parāśara and Garga were known to Varāhamihira (505 A.D).

Āryabhaṭa II says that he is only restating the teachings of "*Vṛddha*" Āryabhaṭa (*MS* Ch.XIII,14).14). Sudhākara Dvivedi in his modern commentary on *MS* remarks that "*Vṛddha*" Āryabhaṭa . is not the same as the well-known Āryabhaṭa I (476 A.D). However, S.R.Sarma comments that the argument given therefore by Dvivedi is not itself conclusive.

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3. "Ayanagraha" and the Precession of Equinoxes

It is interesting that Āryabhaṭa II mentions "*Ayanagraha*" and the number of revolutions it executes in a *Kalpa* (432×10^7 years). He gives the figure as 578159 *bhagaṇas*, in other words $173''.4577$ a year. This is far too high a value, for the annual rate of the precession of the equinox. But the fact that Āryabhaṭa II knows about the motion of the equinoxes precludes the possibility of his being earlier than Brahmagupta. As regards the equinox and its precession, it must be pointed out that Āryabhaṭa II does not mistake the "*Ayanagraha*" for the vernal equinox or a solstice. For him the point represented by the *krānti* (declination) of the *ayanagraha* is the equinox and hence taken oscillatory.

MS describes a method of calculating the *degree* of precession (III.13), but the equinoctial motion as calculated from it is not found always constant, but varying considerably. *Rājamṛgāṅka* (1042 A.D) has a constant motion for all times. From this, it appears that Āryabhaṭa II lived before the equinoctial motion was correctly known.

S.B.Dixit (1969, 1981) comes to the conclusion that Āryabhaṭa II lived about 953 A.D (śaka 875) Datta and Singh (2001) place him at 950 A.D in their *History of Hindu Mathematics*.

4. Some Highlights of Mahasiddhānta

(i) Main contents

The text comprises 18 chapters in two parts: *Pūrvagaṇita* and *Golādhyāya*. The former has 13 chapters in which the concepts and procedures of astronomy proper are discussed.**gōḷādhyāya**, in five chapters deals with cosmography and mathematics proper – arithmetic, algebra etc. The last chapter is devoted exclusively to *Kuṭṭakāra*. This last and elaborate topic deals with the first order $(ax+by=c)$ the second order (Nx2 + 1 = y^2) ***indeterminate equations***. In fact Āryabhaṭa II makes improvements over the earlier known methods.

(ii) Letter Numerals - - Kaṭapayādi

It is well known that, even like many other disciplines, the Sanskrit compositions on mathematics and astronomy have been in the poetic form i.e. in *ślokas* (verses) since symbols for numbers cannot be used. These were replaced by either word–numerals (*bhūta saṅkhyā*) or letter–numerals. In both these forms we have combinations of letters providing ample choice for the composer to fit them into the chosen meters (*chandas*). Most of the composers of texts (especially from the north) on mathematics and astronomy adopted the *bhūta saṅkhyā* system. However, the composers particularly from the Kerala region have preferred a particular form of letter–numerals called "*Kaṭapayādi*". Independently of this practice, Āryabhaṭa I (476 A.D) adopted his own letter–numeral system. But this practice was not continued, may be due to the inconvenience of pronunciation of the resulting letter combinations and also due to less choice.

Āryabhaṭa II has adopted a modified *Kaṭapayādi* system. Besides some minor changes regarding the use of vowels and consonants, in this system the numerals – represented by letters – are read from *left to right*. This is opposite to the usual practice of reckoning them from *right to left* (*aṅkānām vāmato gatiḥ*).

(iii) Area of a Cyclic Quadrilateral

Brahmagupta (628 CE) states that the "exact" (*sūkṣmaṃ*) area of a (cyclic) quadrilateral is "the square-root of the product of four sets of half the sum of the sides (respectively) diminished by the sides". In his own words,

bhujayogārdha catuṣṭaya bhujonaghātāt padaṃ sūkṣmaṃ |

– *Br.Sph.Śiddh.XII*, 21

If *a,b,c,d* are the sides of a cyclic quadrilateral *ABCD* and $2s$ is their sum, then

$$\text{Area of } ABCD = \sqrt{(s-a)(s-b)(s-c)(s-d)} \quad \text{where} \quad s = \frac{(a+b+c+d)}{2}.$$

This formula was "rediscovered" in Europe nearly a thousand years later by W. Snell in 1619 CE. !

However, Brahmagupta missed mentioning *explicitly* that the formula is exact only for a "cyclic" quadrilateral. However, from the context it is clear that the discussions are relevant only to cyclic quadrilaterals.

Brahmagupta's successors, like Āryabhaṭa II and Bhāskara II, mistook that the formula was meant for *any* quadrilateral. When they got inaccurate answers, they found fault with Brahmagupta.

In fact, Āryabhaṭa II does not hesitate to use the words, *"fool or devil"*, of course without mentioning Brahmagupta! He says

yo'sau mūrkhaḥ piśāco vā |

– *MS, XIV*, 70.

This derogatory reference sounds as though it is a retort to an unwarranted criticism Brahmagupta himself has hurled against his celebrated predecessor, Āryabhaṭa I (b.476 CE) for his departure from tradition. In fact Brahmagupta, amidst his frequent invectives, comments: " Āryabhaṭa knows nothing – in mathematics, time and (celestial) spherics":

jānātyekamapi yato nāryabhaṭo gaṇita – kāla – golānāṃ |

– *Br.Sph.Śiddh.XI*, 42.

5. Mean Motion of Planets - *Parāśara – Siddhānta*

The entire second chapter of *MS* is devoted to the mean motions and positions of the planets. Āryabhaṭa II mentions that the related parameters are all as given in the *Parāśara Siddhānta*. Even in later chapters the author refers to the Parāśara's system.

Parāśara is known to Varāhamihira but no astronomical work of his is available. Interestingly, MS acknowledges *Parāśara Siddhānta* as giving the equinoctial motion (of the *ayanagraha*) in a *Kalpa* (432×10^7 years) as 571709 revolutions. This gives the annual rate of mean motion as $174''.5127$ a year. Obviously this is not the rate of precession of the equinoxes (about $50''.26$ per year).

S.R. Sarma concludes that if *Parāśara Siddhanta*, as known to Āryabhaṭa II, really mentions equinoctial motion, then this Parāśāra must be later than Brahmagupta and not the same as the one mentioned by Varâhamihira. This means that the *upper limit* for Āryabhaṭa II's date is the seventh century. As mentioned earlier, the middle of the tenth century appears to be a good estimate for his date.

6. Conclusion:

In the present article we have provided a brief introduction to Āryabhaṭa II and his work. A detailed presentation of the contents of *MS* is presented by S.R.Sarma in his Ph.D thesis (1966). We are indeed highly indebted to S.R.Sarma and his thesis.

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Bhāskarācārya - II

The Most Popular Indian Astronomer

Dr. S.Balachandra Rao[1], Dr. Padmaja Venugopal[2] and Dr. S.K.Uma[3].

Yeṣāṃ sujātiguṇa var *ga vibhūṣitāṅgī*
s′uddhākhilavyavahṛtiḥ khalu kaṇṭhasaktā |
līlāvatī iha sarasoktim udāharantī
teṣāṃ sadaiva sukha sampad upaiti vṛiddhim ||

"There is always happiness, wealth and prosperity to those around whose neck a chaste and pure lady, *Līlāvatī*, belonging to a respectable family, endowed with good virtues, throws her arms" - *Līlāvatī* of Bhâskara II.

This is the *s′loka* with which Bhāskarācārya, the most popular among Indian mathematicians and astronomers, concludes his popular text *Līlāvatī*. But then anyone wonders how this stanza, glorifying pleasure and fortune of one who is blessed with the grace of a beautiful and virtuous lady, be related to mathematics. This *s′loka* is a double entendre (*s′lesa*) having another meaning : "Joy and happiness are indeed ever increasing in this world for those who have the text of *Līlāvatī* clasped to their throats (i.e., mastered by them) decorated as the numbers are with (the mathematical topics) of neat reduction of factors, multiplication and involution, pure and perfect as are the solutions (of the problems), and tasteful as is the speech which is with examples".

Bhâskara's greatness lies in making mathematics highly irresistible and attractive. Our celebrated mathematician Bhâskarâcârya, of the twelfth century, is generally referred to as Bhâskara II to distinguish him from his namesake of the seventh century.

1. Bhâskara's time and works

According to Bhāskarā's own statement, he belonged to Vijjaḍa Viḍa (or Bijjaḍa Biḍa) near the line of Sahyādri mountains. He was born in 1114 A.D. Bhākara's father was mahēśvara saintly and scholarly person belonging to the *S′āṇḍilyagotra*.

The place Vijjaḍa Viḍa is identified with modern Bijapur belonging to Karnataka. However, some scholars have identified the place with other places in Maharashtra.

Bhâskara's celebrated work, *Siddhānta S′iromaṇi* consists of four parts namely *Līlāvatī*, *Bījagaṇitam*, *Grahagaṇitam* and *Golādhyāya*. The first two, treated as independent texts, deal exclusively with mathematics and the last two with astronomy.

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In 1183 A.D., when he was 69 years old, Bhâskara composed another smaller astronomical text called *KaranaKuṭuhalam*.

Bhâskara has given the *ayanāms'a* (the amount of the precession of the equinoxes) as for the *s'aka* year 1105 (i.e., 1183 A.D.) when he composed his *karaṇa* work.

A stone inscription was discovered at a place called Pātan, about 10 miles southwest of Chalisgaon in Maharashtra. According to that inscription (see *Epigraphia Indica*, Vol. I, pp. 340), Changadeva, a grandson of Bhâskara II, was an astronomer at the court of King Singhana of the Yadava dynasty. King Singhana ruled at Devagiri from *s'aka* 1132 to 1159 (i.e., 1210 to 1237 A.D.). Changadeva built a monastery at Pâtan for propagating the works of Bhāskarācārya and his descendents. King Saideva of Nikhumbha dynasty made an endowment for the maintenance of the monastery in *s'aka* 1129 (i.e., 1207 A.D.).

Changadeva, is his inscription, states that King Jaitrapâla invited *Lakṣmīdhara*, son of Bhâskarâcârya from the town Pâtan.

Līlāvatī is an extremely popular text dealing with arithmetic, elementary algebra, geometry and mensuration. *Bījagaṇitam* is a treatise on advanced algebra.

Grahagaṇitam and *Golādhyāya* are completely devoted to computations of planetary motions, eclipses etc., and rationales of spherical astronomy.

Bhāskara has condensed in his mathematical texts the remarkable contributions of his predecessors, Āryabhaṭa, Brahmagupta, Srīdhara and Padmanâbha. Although Bhâskara does not mention the ninth century Karnataka mathematician Mahâvira's name, he seems to be greatly influenced by the latter's work. Many examples given by Bhâskara greatly resemble similar examples given in the *Gaṇita Sāra Saṅgraha* of Mahāvīra. The coincidence cannot be just accidental especially since Mahāvīra preceded Bhâskara by nearly three centuries and hailed from almost the same region.

Bhâskara has written a detailed commentary on the *Siddhānta S'iromaṇi* and it is called *Vāsana Bhāṣya*. In this commentary very interesting and illustrative examples are worked out.

Bhâskara mentions that his birth took place in the *s'aka* year 1036 (i.e., 1114 A.D.) and that he composed his *Siddhānta S'iromaṇi* when he was 36 years old (i.e., in 1150 A.D.)

In the *Grahagaṇitam* part, Bhâskara has extensively dealt with the determination of mean and true positions of planets, the three problems ("*tripras'na*") relating to time, direction and place, the lunar and solar eclipses, risings and settings and conjuctions of the planets.

The chapter on spherical astronomy, *Golādhyāya*, is very important from the point of view of theoretical astronomy. Rationales for the formulae used are provided. The eccentric and epicyclic theories for the motions of planets, as theoretical bases, are clearly developed.

An account of the large number of astronomical instruments is given in *Yantrādhyāya*. Bhāskara greatly improved upon the formulae and methods adopted by earlier Indian astronomers.

In the *KaranaKuṭūhalam*, Bhāskara has adopted as epoch, for computations, the sunrise of February 24, 1183 A.D. (Julian), Thursday. This tract is also well-known as *Garhāgama Kuṭūhalam*. Some almanac-makers are using this text even now for their computations. In fact a voluminous work called *Jagaccandrikā Sāranī* consists of ready-to-use tables based on Bhāskara's tract. The text of *Karana Kuṭūhalam* consists of 139 *s′lokas**.

The work *Siddhānta S′iromaṇi* of Bhāskara has attained the highest degree of excellence, among the Indian astronomical treatises, mainly because of various simplified methods and rationales for the underlying theories. This lucid but detailed treatment extends starting from the computation of *ahargaṇa* upto abstruse questions like those of parallax and the sine tables. Among all the *Siddhāntic* texts, Bhāskara's *Siddhānta S′iromaṇi* merits as the best and exhaustive text for understanding Indian astronomy.

2. Corrections to mean positions of planets

In finding the true positions of planets the earlier astronomers had recognised the following important corrections to be applied to the mean positions :

(i) *Des′āntara samskāra* - Due to the difference in longitudes of the given place and the central meridian (Ujjayinī); there is difference in the timings of the sunrise on the same day at places with different longitudes.

(ii) *Cara samskāra* - Due to the difference between the latitude of the given place and the latitude of Laṅkā (i.e., the equator).

(iii) *Bhujāntara samskāra* - When the above two corrections are effected, we get the mean position of a planet at the *mean midnight* of the given place. But the *true midnight* at the place differs from the mean midnight by what is called "equation of time". This equation of time is made up of two constituents - one due to the *eccentricity* of the earth's orbit and the other due to the *obliquity* of the ecliptic with the celestial equator.

(iv) *Udayāntara samskāra* - As pointed out earlier, this is the correction to get the positions of planets at the *true* midnight or sunrise caused by the inclination (obliquity) of the ecliptic with the celestial equator.
Although it was astronomer S′rīpati (1025 A.D.) who gave this *udayāntara* correction for the first time and actually called it "*yātāsava*", Bhâskara later provided the rationale for this additional correction.

3. Moon's equations

After obtaining the *mean* longitude of the Moon, some important *equations* have to be applied for securing the *true* position. Among hundreds of such corrections to be applied to the mean position, the following are the three most important equations. Their approximate coefficients are also given according to modern astronomy (see Brown's *Lunar Theory*).

(i) **Equation of Centre** (*Mandaphala*)

$$\text{Equation of centre} = \left(2e - \frac{1}{4}e^3\right)\sin\Phi = (377'\,19''.06)\sin\Phi$$

***See:*Karanakûtuhalam* of BhaskaracâryaII-An English Translation with Notes and appendices, Dr S Balachandra Rao and Dr S K Uma, IJHS., Indian National Science Academy, New Delhi,2008.**

where e is the eccentricity of the Moon's elliptical orbit and Φ is the Moon's mean anomaly given by

Φ = (Moon's mean long. – perigee)

Note : In Indian astronomy, instead of perigee, its opposite point, **apogee**(*mandocca*) is considered.

(ii) **Evection** :

$$\text{Evection} = \frac{15}{4} me \sin(2\xi - \Phi) = (76'\ 26'')\sin(2\xi - \Phi)$$

where x is the elongation of the Moon from the Sun

i.e., x = (Mean long. of the Moon – Mean long. of the Sun) and m is the ratio of the mean daily motions of the Sun and the Moon.

(iii) **Variation** :

$$\text{Variation} = \left[\frac{11}{8}m^2 + \frac{59}{12}m^3 + \frac{893}{72}m^4\right]\sin(2\xi) = (39'\ 30'')\sin(2\xi)$$

Thus, considering only these three important equations of the Moon, the (approximate) true longitude () of the Moon is given by

$$\lambda = L + 377'' \sin\Phi + 76' \sin\left(2\xi - \Phi\right) + 39'.5 \sin(2\xi)$$

where L is the mean longitude of the Moon. In particular, at the *syzygies* (i.e., the new moon and full moon), x = 0 or x = 180º in which case the variation term vanishes and the evection term reduces to $-76'\sin\Phi$. In that case, the (approximate) true longitude of the Moon at the new or full moon is given by

$$\lambda = L + 377' \sin\Phi - 76' \sin\Phi = L + 301' \sin\Phi$$

The equation of centre (*mandaphala*) was known since even before Āryabhaṭa -I (476 A.D.). In fact, Āryabhaṭa himself gave the coefficient in the correction term as 300¢ 5². Brahmagupta in his *Uttara Khaṇḍ Khādayaka* gives it as 301¢.7.

Actually, the second equation of the Moon viz., evection (combined with a part of the equation of centre) was first given, among the Indian astronomers, by Mañjula (or Mañjula, 932 A.D.) in his *Laghumânasa*. Sengupta points out, **"In form the equation is most perfect, it is far superior to Ptolemy's; it is above all praise."** Bhāskara-II gets the credit of being the first among the Hindu astronomers in introducing the Moon's equation which is now called evection into a *siddhântic* text. It is remarkable that Bhāskara's discovery preceded that in the west (by Tycho Brahe) by nearly four centuries.

Apart from the three major equations of the Moon, there is another important fourth equation called, the **annual equation**. The credit of the discovery of this lunar correction, among Indian astronomers, goes to the Orissa astronomer, **Candraśekhara Simha Sâmanta** (19th Century). It is noteworthy that Candra śekhara discovered this important correction independently since he was trained in the orthodox Sanskrit style and totally ignorant of English education or the western development of astronomy. In fact, Candraśekhara's fourth equation works out to be

Annual correction = (11′ 27″.6) sin (Sun's anomaly)

Tycho Brahe took the coefficient wrongly as 4′ 30″ while Horrock's (1639 A.D.) value is 11′ 51″.

4. Cakravâla method to solve $Nx^2 + 1 = y^2$

Brahmagupta (628 A.D.) has the unique honour, in the history of world mathematics, of discovering the general method of solving a second-order indeterminate equation, Varga Prakṛti of the form $Nx^2 + 1 = y^2$ by his *Bhāvana* method.

Bhāskara II improved upon Brahmagupta's method in his *Cakravâla* (cyclic) method. Bhāskara's method dispenses with the necessity of seeking a "trial solution", to start with, for the equation.

The *Cakravâla* method is essentially as follows :

$Nx^2 + K = y^2$ when $K = \pm 2$, or ± 4

We can find *a* and *b* such that $Nx^2 + K = b^2$ for any suitable *K*. We also have N. $1^2 + (m^2 - N)m^2$. Applying the *Samâsa Bhāvana* of Brahmagupta, we readily obtain

$$N\left[\frac{am+b}{K}\right]^2 + \frac{m^2 - N}{K} = \left[\frac{bm+Na}{K}\right]^2 \qquad \text{......(*)}$$

By the *kuṭṭaka* method, choose *m* such that is divisible by *K*, where *m* is suitably chosen so as to make numerically small. Let

$$\frac{am+b}{K} = a_1, \frac{m^2 - N}{K} = K_1 \text{ and } \left[\frac{bm+Na}{K}\right]^2 = b_1.$$

Then, we have

> *Bhâskara's theorem* 1 : When a_1 is an integer, then b_1 and K_1 are also integers.

Equation (*) takes the form $Na_1^2 + K_1 = b_1^2$ where a_1, K_1, b_1 are integers. Now using a_1, b_1, K_1 instead of *a*, *b*, *K*, the process is repeated. Let the new set of integers thus obtained be a_2, b_2, K_2, so that $Na_2^2 + K_2 = b_2^2$. The process is **repeated successively**.

> *Bhâskara's theorem* 2 : After a finite number of iterations, two integers α and β can be obtained such that
>
> $N\alpha^2 + \lambda = \beta^2$ where l= ±1 or ±2 or ±4 .

Thus, starting with $Na^2 + K = b^2$, where *K* is any convenient integer, we can arrive at a solution (a, b) of the equation $Nx^2 + \lambda = y^2$

where λ takes the value 1 or 2 or 4 with either the positive or the negative sign.

Once this solution is obtained, Brahmagupta's usual method will lead to an integral solution of the given equation, $Nx^2 + \lambda = y^2$.

While Bhāskara's first theorem has been proved by Datta and Singh and also by the famous German mathematician Hankel, the proof of Bhāskara's second theorem has been given by A.A. Krishnaswami Ayyangar (see *Jour. Ind. Math. Soc.* Vol. 18 (First Series), Second part, 232-245).

Krishnaswami Ayyangar has also shown that Bhâskara's *Cakravâla* method requires less number of steps than the modern **Euler-Lagrange method** of solving a Varga Prakṛti equation

> *"It (Bhâskara's **Cakravâla** method) is beyond all praise : It is certainly the finest thing achieved in the theory of numbers before Lagrange".*
> - Hankel, the famous German mathematician.

Considering the equation $61x^2 + 1 = y^2$, as an example, Bhāskara II obtains the solution :

x = 226 153 980, y = 1766 319 049

In fact, these are the least non-trivial integral values of *x* and *y* (having 9 and 10 digits respectively) satisfying the equation $61x^2 + 1 = y^2$.

***Note*:** There is an interesting history behind this very particular equation, $61x^2 + 1 = y^2$. The famous French mathematician, Fermat, in 1657 A.D., proposed the above equation for solution, as a challenge, to Frenicle and other fellow-mathematicians. None of them succeeded in solving the equation in integers. But the very same equation, though coincidentally, was completely solved by Bhāskara II about five hundred years earlier.

5. Bhâskara II on differentials: Bhâskara II introduces the concept of instantaneous motion (*tâtkâlika gati*) of a planet in the chapter on true positions of planets (*Spaṣṭādhikāra*) of his *Siddhānta S'iromaṇi*. He clearly distinguishes between *sṭhūla gati* (gross or average velocity) and *sūkṣma gati* (accurate velocity) in terms of differentials.

If *y* and *y'* are the mean anomalies of a planet at the ends of consecutive intervals, then according to Bhâskara,

$$\sin y' - \sin y = (y' - y)\cos y$$

which is equivalent to the result (in our modern notation) :

$$d(\sin y) = \cos y dy$$

In Bhâskara's own words :

Bimbārdhasya koṭijyā juṇastrijāḥarah phalaṃ dorjyāyorantaraṃ |

"The product of cosine of the semi-diameter by the element of the radius gives the difference of the two sines." However, much before Bhâskara, nearly two centuries earlier, Mañjula (932 A.D.) has given the same idea in his *Laghumânasam*. Mañjula uses the fact that the tabular difference of sines for an arc are proportional to the cosines.

Bhāskara II goes further to state that the derivative (taken as a ratio of differentials) vanishes at a *maxima*. He says:

yatra grahasya paramạmphalam tatraiva gatiphalabhāvena bhavitavyam |

"Where the planet's motion is maximum, there the fruit of the motion is absent (i.e., stationary)."

6. Cubic and biquadratic equations

The solution of cubic and higher order equations was a favorite topic in algebra dealt with by the medieval Indian mathematicians.

Bhāskara II gives the solutions of *cubic* and *biquadratic* equations in his *Bījagaṇitam* :

1. Solve the *cubic* equation $x^3 + 12x = 6x^2 + 35$

Solution : The equation can be written as $x^3 - 6x^2 + 12x - 8 = 27$ or $(x-2)^3 = 3^3$

so that $x - 2 = 3$ or $x = 5$.

This is the only real root.

2. Solve the *biquadratic* (i.e., fourth degree) equation $x^4 - 2x^2 - 400x = 9999$

Solution : Adding $4x^2 + 400x + 1$ to both sides, we get

$x^4 + 2x^2 + 1 = 4x^2 + 400x + 10{,}000$

or $\left(x^2 + 1\right)^2 = (2x + 100)^2$

or $x^2 + 1 = 2x + 100$

i.e., $x^2 - 2x + 1 = 100$ or $(x - 1)^2 = 100$

so that we get

$x - 1 = 10$ or $x = 11$.

The other roots, having *complex* values, are not considered since the idea of complex numbers was introduced many centuries later by the European mathematicians.

7. Conclusion

In this present article we have provided an introduction to the life and works of the popular Indian mathematician and astronomer, Bhâskara II his works actually go deeper into the concepts and procedures. The titles listed in the bibliography provide the real insight into Bhâskara's contributions.

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Pavuluri Mallana

Outlines of His Life and Work

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Prologue:

Any reference to Ancient Indian Science, in general, and mathematics, in particular, usually takes us to places like Pataliputra, Ujjain, or Varanasi. A notable exception is a sleepy, small town called Manyakheta (presently known as Malkhed) of Karnataka. During the 9th century A.D, Amogha Varsha Nripathunga (814-880 A.D), of the Rashtrakuta Dynasty, ruled the northern part of Karnataka as an emperor. He not only patronized artists, sculptures, and *vaidyas*, but also scholars, poets, and scientists. Among these scientists was a brilliant Jain mathematician by the name Mahaviracharya. He was a much respected mathematician who flourished in Karnataka. He was considered famous much earlier to the great Bhaskaracharya (12th Century) and later to the well-known Brahmagupta (6th Century).

Mathematicians of ancient India were mostly well-versed scholars in Jyothisha and Astronomy. Amongst them, Mahavira shined as a brilliant and exceptional star. He was primarily a mathematician par excellence and not an astronomer or a jyothishka. His magnum opus, *Ganita Sara Sangraha (GSS)*, was a work of high quality for his times. It is unfortunate that mathematicians who succeeded him did not make any reference to either Mahaviracharya or his work, GSS. The reason could be anything.

Mahaviracharya was an ardent Jain. This might have been a reason that Hindu mathematicians of contemporary/later periods eclipsed him. Maybe due to the lack of communication systems, the fame of this mathematician and his work could not reach northern parts of the country like Ujjain, Varanasi, or Pataliputra. However, the fact remains that Mahaviracharya has been one of the greatest mathematicians of ancient India and his monumental work GSS was a famous compendium of mathematics, of his times, free from astronomy or astrology.

Pavuluri Mallana (11th Century A.D):

Pavuluri Mallana lived in the 11th Century A.D. and adorned the court of the Eastern Chalukyan King Raja Raja Narendra of the Vengi Kingdom (1018-1061). He ruled with Rajamahendravaram (now known as Rajahmundry) as the capital. Rajahmundry is built on the banks of the river Godavari in the coastal region of Andhra Pradesh. Little is known about Mallana's early life and about his complete works, except the first three chapters of his book known as *Sara Sangraha Ganitamu* (SSG).

There are two views about the time period of Mallana. One view is that he was a contemporary of the illustrious Adikavi Nannaya Bhattaraka, who wrote the first *Kavya* of Telugu literature. This is the famous translation/trans-creation of the original Sanskrit Mahabharata of Sage Veda Vyasa. Every Telugu speaking person knows the stature of Nannaya and the pre-eminent place that has been given to him and his work (Andhra Maha Bharatam – the two and a half Cantos – Aadi Parvam, Sabha Parvam, and a part of Aranya Parvam). Nannaya was the chief poet in the court of King Raja Raja Narendra. Nannaya was more than a poet; he was a friend, a guide and a philosopher to the king. Mallana was a close associate and friend of Nannaya. Mallana enjoyed along with Nannaya the royal patronage. Obviously, Mallana should have known very well the writings of Nannaya, the style of his translational methods, and the art of composing Telugu poems in different meters. Eloquent evidence to these facts is noticeable in many common features that we find in the writings of both. Some of the common features are listed below.

1. Nannaya translated into Telugu from the Sanskrit original of Sage Veda Vyasa
   Mallana translated into Telugu from the Sanskrit original (GSS) of Mahaviracharya.
2. Both the renderings are not mere word to word translations, but they are trans-creations.

3. Nannaya's opening poem is in Sanskrit – Sardoola Vikriditam Meter.
Sreevanee...
Mallana's opening poem is also in Sanskrit and in the same meter.
Sreekantam...
We note here that both were writing Telugu works but the start was with the Sanskrit slokas.

4. Nannaya concluded his opening sloka with the words
Sreekandharaashraeyasae...
Mallana concluded with the words
Vande Shivam Shraeyasae...

5. Nannaya described The Mahabharata as *Bharata Bharatee Samudramu...* and declared that even Lord Brahma cannot swim across that vast ocean.
Mallana with more assertiveness declared that
Abhinava Sankhyamani Deepti Sara Sungraha Ganita Samudrambu Daruva Ganagiti Preetin...

6. Nannaya said that he would render his translation in the way that was possible for him.
Mallana with more confidence said
Ganitamu Tenuguna Gavimpaga Ganagithini Sukavi Malluda...

7. Nannaya presented his work as a *Champu Kavya* that is an admixture of prose and poetry.
Mallana also did likewise.

8. Nannaya used several types of poetical meters – prosodic structures, and a particular mention need be made on the meter of *Mattakokila* form.
Mallana also used *Mattakokila* poems.

9. Nannaya was a trend setter for Telugu writing as he was the first poet in Telugu literary composition.
Mallana wrote the first treatise on a scientific topic like mathematics simultaneously with Nannaya and also independently.

10. Nannaya had a little freedom in his translation of Mahabharata which is a *Kavya*.
Mallana had no freedom at all as his subject theme was mathematics. In contrast we can say that Nannaya had a lot of facility and Mallana had a lot of restrictions.

11. Nannaya had plenty of opportunities to display his poetical skills, use effective figures of speech, and describe scenes with sweet diction and splendid imagery.
Mallana could not afford to have this freedom. He had inbuilt difficulty. However, he had profound knowledge and writing skills in both Sanskrit and Telugu, in their literary traditions and writings. Besides, far reaching and exceptionally good scholarship in his subject namely mathematics flourished up to his times.

12. Nannaya composed the first *Kavyam* in Telugu.
Mallana wrote the first treatise of mathematics in Telugu.
Taking into consideration the state of Telugu language, limitation of the available mathematical knowledge, and the poor communication systems, one cannot but admire Mallana for his venture in successfully bringing out a text on mathematics.

13. Nannaya acknowledged that he was rendering into Telugu Vyasa's original, but Mallana never mentioned a word about the original GSS or its author Mahaviracharya.

Mallana's Family Background:

Pavuluru is a small village even now flourishing, in the Bapatla area of Guntur district. Mallana I, the son of Sivvana I, hailed from a family of village officers (Karanams). Mallana I had four sons, one of them was again Sivvana II. Sivvana II had a son by name Mallana II. Thus, there were two Sivvanas and two Mallanas.

Sivvana I → Mallana I → Sivvana II → Mallana II

Mallana I was the distinguished contemporary of Nannaya and was in the *Asthana* of King Raja Raja Narendra. Mallana I was a great Jyothishka and a poet too in his own right. The King would have gifted him the village *Navakhandavada*, an *agraharam,* near *Pitapuram* in recognition of his services to the king and the state. One naturally opines that this Mallana I would be the author of *Sara Sangraha Ganitam* or popularly known as Pavuluri Ganitam.

The other school of thought attributes the authorship to the grandson, Mallana II. The strong evidence for this view to gain strength and support of knowledgeable scholars is that some manuscripts of Pavuluri Ganitam (partially available) contain a poem which is crucial and is shown as irrefutable evidence to confirm their viewpoint. Due to lack of further details we are not able to pinpoint which Mallana was the author of Pavuluri Ganitam. Thus, both the views continue to confuse the historians of mathematics until further evidence comes out to settle the issue in favour of either of the Mallanas.

Mahaviracharya and Mallana:

Almost 1000 years before there was no tradition at all of writing texts in Telugu, much less a mathematical treatise. However, in Sanskrit, there was a long tradition of writing texts, whether literary or *sastric*. Thus, the great advantage that Mahavira had was denied to Mallana. Mallana took upon himself the Telugu rendering of Mahaviracharya's GSS composed in prosodic Sanskrit (*Champu Kavya* form). GSS is the most voluminous work belonging to the ancient period. This is a milestone in the history of Indian mathematics. It is a compendium of all mathematics known at the time of writing GSS (850 A.D.) setting right the vagueness in the mathematical concepts and refining the ideas already known. It contains 1131 Slokas and about 1000 problems and is presented in eight chapters, captioned *Vyavaharas*. Anka Ganitam, Bija Ganitam, Kashetra Ganitam, Vyavahara Ganitam, geometrical progressions and combinations are found in the text.

Mahaviracharya admitted with humility that what he had done was only picking up some valuable material as much as possible for him by diving once or twice into the mighty ocean of mathematics. In his own words:

Kinchi dudh dhruthya tatsaram vakshyehammatisaktitah
Alpagrandhamanalpartham ganitam sara sangraham

Mahavira was a devote Jain and he displayed an abundance all through his writings, wherever possible, about his attachment and devotion to the religion.

Mallana composed in *Champu* form and he was endowed with sound knowledge of Sanskrit, Telugu, and Mathematics. Mallana was a staunch Saivite. He disliked the Jain atmosphere of the GSS and eventually did not make a mention of the fact that he had translated GSS – a highly unethical act on his part.

It is a fact that SSG of Mallana was not a translation, *mutatis mutandis.* He omitted several things, added new portions, and framed new problems with full freedom. Most of the problems he had included were his own. In

certain situations, though Mallana had taken a problem from the original, he changed the data to give it a flavour of his own. Some points in comparison and contrast between GSS and SSG can be listed.

1. Mahavira commences his work with a prayer offered to Jinendra:
   Namastasmai Jinendraya Mahavirayatayinae
   Mallana prayed Lord Shiva:
   Tam Shivakaram Vande Shivam Preyasae

2. Mahavira presented his work in eight parts – *Vyavaharas*
   Mallana made it into ten parts:
   Parikarma Ganitamu
   Bhinna Ganitamu
   Prakeerna Ganitamu
   Thrairasika Ganitamu
   Misra Ganitamu
   Sutra Ganitamu
   Kshetra Ganitamu
   Suvarna Ganitamu
   Khata Ganitamu
   Chaya Ganitamu

This is the reason why SSG is also called *Dasa Vidha Ganitam*. However, the work is popularly known as Pavuluri Ganitam as already mentioned. It is our misfortune that all the ten parts are not available. Only the first three chapters are available in print.

3. Mahavira has mentioned the measures *(kolathalu)* that were in vogue during his times. Mallana totally deviated from this scheme as those measurements would have become archaic or out of usage or irrelevant. He chose the signs and measures that were in vogue in Andhra region of his locality. We know that even Mallana's measures are not relevant now. While describing various measures, Mallana said *"Maanamugaa Vyavaharinthru Mahinandhra Janul"* In measuring time Mallana mostly followed Mahavira, except in the following items.
   a. 3 Seasons (rituvulu) = 1 Ayanam
   2 Ayanams = 1 Year
   We use even now this terminology, but it is not known why Mallana discarded them. However, he said that 6 Seasons (rituvulu) = 1 Year
   b. Mallana used the words *Veesam* for 1/16, *Paraka* for 1/8, *Paatika* for 1/4, and *Addiga* for 1/2.

4. Mahavira referred to numbers that are powers of ten with names like
   Sasharam – 10^3
   Laksha – 10^5
   Koti – 10^7
   Arbudam – 10^{10}
   Kharvam – 10^{12}
   Mahaakshoni – 10^{18}
   Mahaakshobham – 10^{23}
   He stopped with *Mahaakshobham* and thus he had given names for 24 place values.
   Mallana extended the scheme up to 36 place values like:
   Nidhi – 10^{24}
   Paratam – 10^{26}
   Mahabhuri – 10^{29}
   Bahusam – 10^{33}
   Sagaram – 10^{35}

5. Mahavira used, at times, symbolic words to represent numbers which has significance to the Jaina religion.
Ratna for 3 (triratnamulu)
Kashaya for 4 (krodha, mana, maya, lobha)
Mahavira was not averse to use the symbolic words having relationship with Shaiva tradition like
Haranethra for 3
Pura for 3
Kumara Vadana for 6
Rudra, Hara for 11
Mallana totally avoided using such Jaina symbolic words for numbers. Whatever symbolism he used was related to Shaivism. Thus he displayed his intolerance for the Jaina religion.

6. Mahavira commenced every part *(Vyavahara)* of his work with Jaina invocation.
Mallana commenced every chapter (evidenced by the available three chapters) with invocation to Shiva in Sanskrit slokas. (We note that this is a deviation from Nannaya's scheme).

7. Acharya (Mahavira) used clusters of slokas at times to put forth fully the mathematical steps of a certain concept. Mallana abridged them and was content in putting the concept in a single sloka.

8. Acharya commenced the elementary operations of arithmetic with multiplication and not with addition and subtraction as usual. Mallana followed suit. This approach is unique in teaching arithmetic and is found only in <u>GSS</u> and <u>SSG</u>. Mallana adds that as the primordial *Omkara* is the first thing for Vedas, multiplication is the starting point for mathematics.
Amnaayambuluku Pranavambunum Bole,
Sakala Ganitambuluku Prathamakarmambai
Gunakaravikhyathambagu Ganukarambu

9. Acharya described the elementary operations with the number zero and committed a blunder by defining $(x \neq 0) / 0 = x$. Mallana avoided this result perhaps judiciously when he defined the operations with the number zero as follows.
Sunnayu Sunnayu Penchina
Sunnaya; Tatkriti, Ghanambu Sunnaya Vachhun
Sunnayu Lekkayu Penchina
Sunnaya Tanamari Yundu Susthira Reethin
Mallana avoided giving meaning to operations like "0 and ^{3}"0, which were dealt with correctly by Acharya.

10. Acharya displayed sound knowledge of divisibility rules, which Mallana also had. This is noticed vividly in the *Kantahara* number problems. In this section, Mallana excelled Mahavira, in setting up very interesting and original number problems.

11. In squaring numbers, Acharya described four methods. Mallana took the last of the four methods, which was good enough for the purpose of squaring.

12. In the case of computing cubes of numbers, <u>GSS</u> described many methods, while <u>SSG</u> adapted the last amongst them.

13. To frame a problem involving unit fractions in Geometric Progression to find the unknown number, Acharya indulged in unnecessary, unwanted, and irrelevant amorous descriptions of a couple. Mallana discarded all these steps and came to the point by giving a minimum description of the scenario and directly framing the problem *(Prakeerna Ganitamu)*

14. In another instance, Acharya, while dealing with Arithmetic Progressions, made a mention of a series with fractional number of terms, which is meaningless from practicality, but can have mathematical existence. This was followed by Mallana.

15. The anecdote of a pundit requesting a benevolent king to grant him a simple gift of grains; "Oh King, in 64 squares of the chess board please place grains as 1 in the first square, 2 in the second square, 4 in the third square, 8 in the fourth square, 16 in the fifth square, and so on..... 2^{63} in the 64^{th} square (last square)." The king failed to arrange this gift as it was impossible in time and in material. This problem is not found in GSS, but is found in SSG. Mallana gave a rich collection of such recreational problems of his own in his SSG.

16. After narrating the properties of the elementary operations of numbers with the number zero, Acharya listed the properties of negative numbers. The highlight therein was his reasoning as to why negative numbers do not have square roots. Mallana did not dwell upon this important point. In addition, while discussing Arithmetic Progressions Mallana did not refer to any Arithmetic Progression with negative common difference.

17. Acharya was undoubtedly a creative mathematician while Mallana was mostly a mathematician.

18. GSS contains 1131 slokas and about 1000 problems presented in 8 *Vyavaharas*, Mallana's work, which is available partially, contains 407 ? poems.

19. In dealing with Progressions, both Arithmetic and Geometric, Acharya excelled, in certain instances, the western writers. He introduced the new operations *Sankalitham* and *Vyuthkalitham* in dealing with the series. These two operations are twisted addition and subtraction. They are a little out of the way. Acharya preferred to introduce them after dealing with the operations of multiplication and division. Mallana followed Acharya in rendering translation of the theory portion only and proceeded in his own way in the later parts commendably by framing examples and problems.

20. *Prakeerna Ganitam,* a little advanced stuff in arithmetic needed algebraic methods to arrive at the answers. Acharya was at his best in the exposition of this part and Mallana did more commendably his Telugu rendering. In certain places, he even excelled Acharya.

Epilogue:

Mallana's SSG is the first mathematical treatise in Telugu. If we take into account the nascent state of Telugu language, which was just shaping itself to be a vehicle of literary works, the total lack of mathematical culture in the society and the problem of suitable terminology in the language, Mallana's venture disserves all praise. Though he had before him a readymade matter in Sanskrit, he adopted a methodology, which facilitated him to add or delete or modify concepts, problems, and themes.

It is unfortunate that the tradition inaugurated by Mallana did not continue and flourish. Except for stray writings on mathematics and commentaries on Sanskrit works about mathematics, now and then, there has been no substantial or original contribution. The tradition initiated by Mallana should be revived and revitalised.

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Chandra Sekhara Samanta
-The last Classical Indian Astronomer

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1. Introduction:

The Indian contribution to mathematics and astronomy after Bhāskara II (b.1114 A.D.) somewhat declined, especially in the northern part of India. However, as is now well-known this condition in north India was more than compensated by the rich contribution from Kerala in the south. In fact for about five centuries mathematics and astronomy flourished in Kerala with truly startling contributions like (i) infinite series by Mādhava onwards, (ii) quasi-heliocentric model by Nīlakaṇṭha Somayāji and (iii) calculus related results like the differentials of inverse trigonometric ratios and the 'quotient formula'.

Outside Kerala, there were here and there some great original authors of *siddhāntic* texts as well as commentators. Among them stand out the Maharashtrian astronomer Gaṇes′a Daivajña (working from Varanasi), the author of *Grahalāghavam* (1520) and during the colonial rule, Samanta Chandrasekhar Simha from Orissa. The great Samanta was perhaps the last among our classical Indian naked-eye astronomers.

Samanta Chandra Sekhar is a celebrated traditional Indian astronomer from Orissa. The Samanta, who flourished towards the end of the nineteenth century, was a self-trained astute astronomer who was completely insulated from Western knowledge - both English language and European astronomy.

Samanta Chandra Sekhar - generally called Pathani Samanta - was born on December 13, 1835 in the royal family of the erstwhile princely state of Khandpara in Orissa. From his young age the Samanta developed a great interest in astronomy and mastered traditional Sanskrit texts like the *Sūrya siddhānta* and Bhāskara's *Siddhānta s′iromani*. He fabricated his own instruments for observing the planets and constellations. While the Samanta kept himself busy observing the heavenly bodies since he was 15 years old, it was only when he was around twenty-three years that he started systematically recording his observations. Three years later he composed his treatise in Sanskrit, *Siddhānta darpaṇa*. Prof. Jogesh Chandra Ray of Cuttack College (now Ravenshaw College) played the pivotal role in not only introducing the astronomer Samanta to the world but even in arranging to get published his astronomical treatise, *Siddhānta darpaṇa* (hereafter *SD*). Later, in 1893, Samanta Chandra Sekhar was conferred with the coveted title *Mahāmahopādhyāya* by the British Govt., thanks to the recommendation of Prof. Nyayaratna, Principal of the Sanskrit College, Calcutta. It is truly heart-touching to know that the Samanta had no option but to keep the manuscript of his Sanskrit treatise, written on palm leaves in the Oriya script, for thirty years lying in a corner of his house.

2. Samanta's Innovations:

In *SD* we find a lot of innovations in the astronomical procedures and parameters like

(i) The *bhagaṇas* of the heavenly bodies and important points in a kalpa for determining the mean daily motions,

(ii) The number of civil days in a kalpa which defines the length of a solar year,

(iii) The mean positions of planets at the Kali epoch,

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(iv) The epochal mean positions at the Samanta's chosen epoch viz., April 12, 1869 at the mean sunrise at *Laṅkā*,

(v) The rate of precession of the equinoxes and the year of zero- *ayanāms′a*,

(vi) The inclination of the planets' orbital planes with the plane of the ecliptic (*Krānti vṛtta*),

(vii) Determination of the celestial latitudes of the planets,

(viii) The angular diameters of the sun, the moon and the planets,

(ix) The *paridhis* (peripheries) of the *manda* and *s′īghra* circles and so on.

Conceptually, what draws our attention is the heliocentric-like model of planetary model proposed by Samanta Chandra Sekhar. Although similar models were proposed earlier by Tycho Brahe (1546-1601) and Nīlakaṇṭha Somayāji (1444-1545), it is surely to the credit of the Samanta that he evolved this planetary model independently. Most possibly he was not aware of the models suggested by Brahe or *Somayāji*.

In the present paper we have made an attempt to highlight some computational aspects of the Samanta's text, *Siddhānta darpaṇa* and compared the results with those of modern astronomical procedures as also of the popular *karaṇa text*, Grahalāghavam*. As a test case, the latest lunar eclipse is worked out based on the Samanta's procedure.

3. Revolutions (*bhagaṇas*) of bodies in a *kalpa*

The usual practice, in the traditional *Siddhāntic* texts, of giving the mean daily motions (*madhyama gati*) of the heavenly bodies is through the numbers of revolutions (*bhagaṇas*) executed by these bodies in a long period of a *mahāyuga*$(432\times10^4\ years)$ or a kalpa $(432\times10^4\ years)$. The Sāmanta presents a revised set of the *bhagaṇas* in a kalpa.

In Table 1 we have compared his values (*SD*) with those of the *Sūrya siddhānta* (*SS*) and with our suggested values for the *bhagaṇas*.

Table 1: of bodies in a kalpa ($(432\times10^7$ years)

Bodies	SS	SD	Proposed Modern
Ravi, Budha & S′ukra	4,32,00,00,000	4,32,00,00,000	4,32,00,00,000
Candra	57,75,33,36,000	57,75,33,36,000	57,75,29,85,910
Kuja	2,29,68,32,000	2,29,68,71, 112	2,29,68,76,453
Guru	36,42,20,000	36,41,55,205	36,41,95,066
S′ani	14,65,68,000	14,66,49,716	14,66,56,219
Budha *s′īghrocca*	17,93,70,60,000	17,93,69,67,141	17,93,70,33,867
S′ukra *s′īghrocca*	7,02,23,76,000	7,02,22,57,860	7,02,22,60,402

Footnote : * See *Grahalāghavam* of Gaṇes′a Daivajña ., An Eng. Exposition, Math.Notes etc S. Balachandra Rao and S.K. Uma, INSA, New Delhi ,2006.

We notice from Table 1 that the Sāmanta has rightly modified the *bhagaṇas* of all the five *tārāgrahas* from the ones according to the *Sūrya siddhānta* (*SS*) . While the figure is enhanced in the case of Kuja and S′ani , the same are diminished in the case of Guru, Budha *s′īghrocca* and S′ukra . We have correspondingly a similar behaviour in respect of these bodies in our proposed modern values. In fact, the SD values would have been still closer to the proposed modern values if only the had preferred a better value for the civil days.

In the case of Guru (Jupiter), for example, the motion of the sidereal planet is $10925661''.4$ in a Julian century of 36525 days. Now, for the *mahāyuga* of 1,57,79,07,487 days modern proposed, the *bhagaṇas* of sidereal Jupiter come to 3, 64,195.066. Therefore in a kalpa the**,** by multiplying the above figure by 1000, work out to be 36,41,95,066.

4. *Mandoccas* (apogees) of planets

While a mean planet moves, with uniform angular velocity, along the deferent circle say of radius R, the manda- corrected (true) planet moves along the manda (*vṛtta*) epicycle, say of periphery p. On this epicycle the *manda-ucca* (apogee) U and its geometrically opposite point, *manda- nīca* N are situated. While U is the farthest point, N is the nearest point to the observer.

Most of the ancient Indian astronomers assumed the *mandoccas* of the planets, including that of the sun, as fixed. Only the moon's *mandocca* was considered moving. They were not essentially off-the-mark since the *mandoccas* of planets move too slowly. Again, the ancient and medieval Indian astronomers assigned reasonably a good value for the rate of motion of the moon's *mandocca.* It was only the *Sūrya siddhānta* (*SS*) that prescribed *bhagaṇas* for the *mandoccas* of the planets in a *kalpa*. Chandra Sekhar Simha has rightly noticed that the apogees have a motion, though slow, and given mean rates in items of *bhagaṇas* **.**

The motion of the *mandoccas* of the planets are very small even over a long period. Only in the case of the moon its *mandocca* has a significant rate of motion. The moon's *mandocca* moves at $40^0.676495$ per year according to the *Siddhānta Darpaṇa* and at $40^0.67709$ per year as per modern computation.

5.1 *Manda Paridhi* and equation of centre

As pointed out in section 4, all the planets, the sun and the moon move in their respective *manda* epicycle. The resulting equation is called *mandaphala* which corresponds to the "equation of centre" in the modern heliocentric model. In fact, with a couple of small approximations, the *mandaphala* (MP) is given by

$$MP = \frac{a}{R}\sin(m) \qquad ...5.1$$

where *m* is the *mandakendra (anomaly*), *a* and *R* are respectively the peripheries *(paridhis*) of the *manda* epicycle and the common deferent circle. The ratio *a/R* is taken as that of the two peripheries expressed in degrees. The periphery of the deferent circle is taken as $R = 360^0$, a constant, common to all the bodies. The peripheries *a* have different values (in degrees) for different bodies. While a few traditional Indian astronomers-notably Brahmagupta in his *Khaṇḍa khādyaka* -took the periphery *a* of each

planet as a constant, many others have rightly considered a *variable* periphery a for each body. They have prescribed the minimum and the maximum values for a. For them a is a function of the *mandakendra (*anomaly m). The ranges of variations of the *manda* peripheries for the heavenly bodies according to different texts are listed in Table 2.

Table 2: Manda peripheries

Body	*KK &VSS*	*Āryabhaṭa*	*Sūrya siddhānta*	Modern proposed (Deg)
Ravi	14^0	$13^0.5$	$13^0 40'$ to 14^0	$11.^0 40 - 12^0.31$
Candra	31^0	$31^0.5$	$31^0 40'$ to 32^0	$36^0.81 - 42^0.23$
Kuja	70^0	$63^0 - 81^0$	72^0 to 75^0	$59^0.3 - 74^0.92$
Budha	28^0	$22^0.5$ to $31^0.5$	28^0 to 30^0	$109^0.8 - 184^0.72$
Guru	32^0	$31^0.5$ to $36^0.5$	32^0 to 33^0	$32^0.69 - 36^0.89$
Sukra	14^0	9^0 to 18^0	11^0 to 12^0	$4^0.87 - 4^0.95$
Sani	60^0	$40^0.5$ to $58^0.5$	48^0 to 49^0	$37^0.42 - 43^0.04$

Note: *KK* **:** *Khanda khādyaka*, *VSS* **:** *Varāhamihira's Saura siddhānta*.

In the case of variable peripheries, the left-hand (lesser) values are the values of the periphery a at the end of the *odd* quadrants ($m = 90^0$ or 270^0) and the right-hand (higher) values at the end of the *even* quadrants ($m = 180^0$ or 360^0). Here, m is the *mandakendra*, the anomaly from the *mandocca.*

If a_e and a_{0-} are the peripheries at the ends of even and odd quadrants respectively then the variable periphery a at any instant, as a function of m, is given by

$$a = a_e - (a_e - a_0)\left|\sin m\right|$$

Chandra Sekhar Samanta gives the following mean (*madhyama*) *paridhis* (in degrees) of the *manda vṛttas* of the *tārāgrahas* :

Kuja 69, Budha 27, Guru 34.5, Sukra 12 and Sani 39. The Samanta provides methods for the computation of the true *paridhi* of each planet. These *paridhis* are functions of the *manda* anomaly and hence variable.

We have to appreciate the important point that it is this *variable* nature of the *manda* epicycle that results in the locus of the true planet, atleast as a very close approximation, in an elliptical orbit. Here, Samanta Chandra Sekhar scores over the authors of the *Khaṇḍa Khādyaka* and Varāhamihira's *Saura siddhānta*.

5.2 Manda paridhi $vis-a'-vis$ **modern computation**

The modern expression for the *equation of centre*, considering the first two terms, in the Fourier expansion, is

$$E = \left(2e - \frac{1}{4}e^3\right)\sin m + \left(\frac{5}{4}e^2 - \frac{11}{24}e^4\right)\sin 2m \qquad ...5.2$$

where e is the eccentricity of the elliptical orbit. Since e is generally very small, ignoring the higher powers of e,

the equation of centre, to the first approximation is

$$E \approx (2e)\sin m \qquad ...5.3$$

Interestingly, comparing (5.1) and (5.3) and noting that $MP = E$,

we have

$$a = 2e \quad \text{(both in radians)}$$

However, for better accuracy, we consider the powers of e upto e^4 in (5.2).

$$\text{Let } e_1 = 2e - \frac{1}{4}e^3 \text{ and } e_2 = \frac{5}{4}e^2 - \frac{11}{24}e^4 \qquad ...5.4$$

Then (5.2), using (5.4), can be written as

$$E = e_1 \sin m + e_2 \sin 2m$$
$$= e_1 \sin m + 2e_2 \sin m \cos m$$
$$i.e., E = (e_1 + 2e_2 \cos m)\sin m \qquad ...\ 5.5$$

From (5.5) we observe that the coefficient of $\sin m$ in the equation of centre is a *variable* and a function of the anomaly m. Now, from (5.1) and (5.5) it follows that the ratio of the peripheries a/R of the *manda* epicycle corresponds to $(e_1 + 2e_2 \cos m)$ where e_1 and e_2 are defined in (5.4).

However, we have to point out that the *manda paridhi* assigned to Buddha (Mercury) in our traditional *siddhāntas* does not reflect the high eccentricity of Buddha's (heliocentric) orbit. For example, the *Sūrya siddhānta* has assigned the variabliable value 28^0 to 30^0 to Buddha's *manda paridhi*. But, considering the high eccentricity of Buddha's orbit, its *manda* periphery should vary from 109.8^0 to 184.7^0. Similarly, the *manda* periphery of S′ukra (Venus) would be nearly half of the traditionally assigned range.

6. Civil days in a kalpa

The traditional Indian astronomical text give the number of civil days (*sāvana dinas*) in either a *mahāyuga* (432×10^4 years) or a kalpa (432×10^7 years). In fact, the number of civil days defines the length of a solar year used in a particular text. The civil days in a *mahāyuga* according to different texts are compared in Table3.

Table 3: Civil days in a *mahāyuga*

	Texts	Civil days
1	Āryabhaṭīyam (Āryabhaṭa I)	1,57,79,17,500
2	Khaṇḍa khādyaka (Brahmagupta)	1,57,79,17,800
3	Mahāsiddhānta (Āryabhaṭa II)	1,57,79,17,542
4	Siddhānta s′iromaṇi (Bhāskara II)	1,57,79,16,450
5	Siddhānta darpana (Sāmanta)	1,57,79,17,828
6	Sūrya siddhānta	1,57,79,17,828
7	Proposed modern	1,57,79,07,487

Taking the modern value of the sidereal (*nirayaṇa*) sun's daily motion as SDM = 3548″.1928098 , the length of a sidereal solar year becomes 365.2563627378105 days. Allowing a maximum numerical error of ± 5 in the eighth digit in the value of SDM, correspondingly, the number of civil days in a *mahāyuga* turns on to be 1,57,79,07,487 days as shown in Table 3.

It is not clear as to what prompted the Sāmanta to adopt the *Sūrya siddhānta* value for the number of civil days.

7. Epochal mean positions of SD

Samanta Chandra Sekhar has given the mean positions of the heavenly bodies as also the special points like the *mandoccas* and *pātas* in his SD for his chosen epoch viz., April 12, 1869 at the mean sunrise at *Laṅkā* . These epochal mean positions of *SD* are compared with those according to the *Grahalāghavam* (GL) and the modern computations in Table 4.

In Table 4, the *nirayaṇa* positions of the bodies are reduced to the values with a common *ayanāms′a* viz., the one adopted by the, Govt. of India based on the *Calendar Reform Committee Report*. It is interesting to note that the *ayanāms′a* according to SD is virtually the same as that of the *Ind. Ast.Eph.* computations for the year 1869 AD.

Table4: Mean positions of bodies for 12/4/1869

Body	Grahalāghavam	Siddhānta darpaṇa	Modern
Ravi	$358^0 40'$	$358^0 15'$	$358^0 15'$
Candra	$3^0 35'$	$3^0 20'$	$3^0 09' 36''$
Candra *Mandocca*	$323^0 35' 20''$	$322^0 34'$	$322^0 18' 28''$
Kuja	$151^0 39'$	$151^0 24'$	$151^0 24'$
Budha	$324^0 06'$	$318^0 14'$	$317^0 47'$
S′ukra	$346^0 06'$	$343^0 41'$	$343^0 40'$
Guru	$5^0 40'$	$3^0 45'$	$3^0 0' 0''$
S′ani	$228^0 59'$	$228^0 12'$	$228^0 24'$
Rāhu	$111^0 51'$	$111^0 19'$	$111^0 19'$
Ayanāms′a	$22^0 27'$	$22^0 01' 18''$	$22^0 01' 40''$

From Table 4, we find a remarkable closeness of the *SD* epochal positions to those according to modern astronomy. Therefore a very important stage for the computational veracity of planetary positions according to *SD* is guaranteed.

Earlier we have noticed that the *bhagaṇas* of the bodies in a kalpa also bear a greater closeness to the modern computations. This ensures the next stage of computational efficacy of the *SD* procedure.

8. Mean positions at the Kali beginning

As per the *ārdharātrika* convention of Āryabhaṭa I (b. 476 A.D.), Samanta Chandra Sekhar has adopted the midnight of 17/18, February 3102 B.C. as the beginning of the *Kali* era. In the chapter on mean planets *s′lokas* 52-55 give the mean positions of the bodies at the *Kali* beginning in *liptis* (minutes of arc). In the

following Table 5, these mean positions are in degrees etc. listed and compared with those according to the modern computations as also a traditional text.

From Table 5, we find discouragingly wide variations in the mean positions among the three systems chosen. According to the *Sūrya siddhānta* it is assumed that all the planets, including Ketu (the moon's descending node) were in *conjunction* while the moon's apogee *(Candra mandocca*) was at 90^0 and the moon's ascending node (Rāhu) was at 180^0. It is surprising that Bhāskara II (b. 1114 A.D.) almost conforms to this assumption of the conjunction though with very small deviations.

Table 5: Mean positions at the Kali beginning

Body	Siddhānta s′iromaṇi	Siddhānta darpaṇa	**Proposed Modern**
Kuja	$359^0 03' 50''$	$344^0 52' 48''$	$335^0 37' 25''$
Guru	$359^0 27' 36''$	$22^0 57' 0''$	$73^0 16' 49''$
S′ani	$358^0 46' 34''$	$330^0 50' 24''$	$36^0 24' 51''$
Ravi *mandocca*	$77^0 45' 36''$	$78^0 39' 36''$	$62^0 34' 24''$
Candra *mandocca*	$125^0 29' 46''$	$120^0 36' 0''$	$113^0 06' 28''$
Rāhu	$153^0 12' 58''$	$198^0 33' 08''$	$188^0 47' 28''$

Note: As regards the apogees of the sun and the planets, most of the traditional texts assumed them as constants. However, the *Sūrya siddhānta* and Bhāskara's *Siddhānta S′iromaṇi* recognised that the Ravi *mandocca* moves though slowly.

Samanta Chandra Sekhar has given a go-by to the pet conjunction assumption at the Kali begininning and rightely so. *SD* epochal positions as also the planetary *bhagaṇas* are all reasonably good. But then how come its Kali beginning positions are at variance from the proposed positions as per the modern computations?

The main culprit appears to be the *ayanāms′a* . Our computations, based on the modern known rate of the precession of the equinoxes and the assumption of the zero-*ayanāms′a* in 285 A.D., the mean works out to be $-46^0 34' 52''$.

9. Computation of lunar eclipse

The procedure of computing the circumstances of the lunar eclipse according to the *Siddhānta Darpaṇa* is demonstrated in this section with an example. The lunar eclipse that took place on the night between May 4 and 5, 2004 is chosen for the purpose. The results are compared with those of the modern astronomical computations.

At the mean sunrise at Laṅkā **on May 4, 2004** we have

True sun : $19^0 43'$, True moon : $188^0 26'$, Rāhu : $17^0 59'$;

True daily motion of the moon, MDM: $897'$;

True daily motion of the sun, SDM: $58'$

Instant of opposition ($parv\bar{a}nta$): 49 gh. From the mean sunrise.

At the instant of $parv\bar{a}nta$ **:**

True sun : $20^0 31'$, True moon : $200^0 31' \equiv M$

$R\bar{a}hu$: $17^0 58'$(approx.) $\equiv R$

Sun's diameter (*Ravi bimbam*): $\frac{SDM \times 11}{20} = 31|54\ kal\bar{a}s.$

Moon's diameter (*Candra bimbam*): $\frac{MDM - 7}{25} = 35|36\ kal\bar{a}s.$

Earth's shadow diameter ($Bh\bar{u}cch\bar{a}y\bar{a}$ *bimbam*):

$$\frac{MDM}{7} - \frac{SDM \times 78}{145} = 96|56|34\ kal\bar{a}s.$$

i.e., i.e., $SDIA = 31|54\ kala\bar{a}s,\ MDIA = 35|36\ kal\bar{a}s,\ SHDIA = 96|56|34\ kal\bar{a}s.$

$Vir\bar{a}hu\ Candra$, (Moon-$R\bar{a}$hu) = $3^0 31' 48'' < 90^0$

$\therefore$ Bhuja of $Vir\bar{a}hu\ Candra = 3^0 31' 48'' \equiv \text{Bhuja}(M - R)$

$$\text{Moon's } s'ara = \left[\text{Bhuja}(M-R) + \frac{\text{Bhuja}(M-R)}{16}\right] \times \frac{1}{11}$$

i.e. $S'ara = 20'27''$ North.

$M\bar{a}naikyakha\d{n}\d{d}a$, MNK$= \frac{\text{MDIA+SHDIA}}{2} = 66|16\ kal\bar{a}s.$

Obscuration, $Gr\bar{a}sa$ = MNK $- S'ara$ = 45|49 $kal\bar{a}s$.

since $Gr\bar{a}sa$ > MDIA (i.e.45|49>35|36), the lunar eclipse is total.

(i) Half-duration of the lunar eclipse:

$$Sthityardha = \frac{\sqrt{(MNK)^2 - (\dot{S}ara)^2}}{(MDM - SDM)} \times 60\ gh.$$

i.e. Half-duration of eclipse = $4^{gh} 30.46^{vig} = 1^h 48^m 11^s$.

(ii) Half-duration of totality :

$$M\bar{a}n\bar{a}ntarakhanda,\ MNA = \frac{(SHDIA - MDIA)}{2} = 30|40\ kal\bar{a}s.$$

Half-duration of totality,

$$Mard\bar{a}rdha = \frac{\sqrt{(MNA)^2 - (\dot{S}ara)^2}}{(MDM - SDM)} \times 60\ gh.$$

i.e. Half-duration of *totality* $= 1^{gh}\,38.0565^{vig} = 0^h\,39^m 13^s.356$.

Therefore, we have the following circumstances of the lunar eclipse:

(i) *Spars′a*, the beginning of the eclipse :

$49^{gh} - 4^{gh}\,30.46^{vig} = 44^{gh}\,29.54^{vig} \equiv 17^h\,47^m\,48^s.96$ after sunrise.

(ii) *Nimīlana*, the beginning of totality :

$49^{gh} - 1^{gh}38.06^{vig} = 47^{gh}\,21.94^{vig} \equiv 18^h\,56^m\,46^s.56$ after sunrise.

(iii) *Madhya*, the middle of the eclipse :

$49^{gh} \cong 19^h\,36^m$ after sunrise.

(iv) *Unmīlana*, the end of totality :

$49^{gh} + 1^{gh}38.06^{vig} = 50^{gh}\,38.06^{vig} \equiv 20^h 15^m 13^s.44$ after sunrise.

(v) *Mokṣa*, the end of the eclipse :

$49^{gh} + 4^{gh}30.46^{vig} = 53^{gh}\,30.46^{vig} \equiv 21^h\,24^m 11^s.04$

Summary of the lunar eclipse dated May 4, 2004

The above timings are converted to IST for the purpose of comparison with the timings of modern computations.

Table 6: Circumstances of Lunar Eclipse on 4/5/2004

	Circumstances	*Siddhānta darpaṇa*	**Modern (Indian Ephemeris)**
1	*Spars′a*	$24^h 14^m 49^s$	$24^h 10^m$
2	*Nimīlana*	$25^h 23^m 46^s.56$	$25^h 20^m$
3	*Madhya*	$26^h 03^m$	$26^h 01^m$
4	*Unmīlana*	$26^h 42^m 13^s.44$	$26^h 36^m$
5	*Mokṣa*	$27^h 51^m 11^s.04$	$27^h 47^m$

We observe that the timings of the lunar eclipse according to *SD* are quite close to those of the modern computation (as giving in the Indian Ephemeris). However, we acknowledge that the timings according to *SD* are w.r.t the *mean* sunrise. If the true sunrise is considered there will be uniform variation, though small, from the modern values in IST.

10. Corrections for bodies' positions

In the Indian astronomical tradition, the practice of introducing *bījas* (corrections) to the parameters has been in vogue for long. The Indian astronomers were aware that the values of the governing parameters, given by them, would be valid only for a century or so and that future competent astronomers should provide further improvements.

For example, the celebrated Kerala astronomer, Parames′vara states:

Kālāntare tu samskāras′cintyatām gaṇakottamaiaḥ

"In the course of time, the (necessary) corrections must be decided by the expert mathematicians". In fact, in his extensive work on computations of eclipses - *Grahaṇamaṇḍana* - observes in all humility that the times of contact etc. of an eclipse as given by him *may at times* differ *slightly* from observed positions: .

10.1 Moon's equations

In computing the position of the Moon, according to the *siddhāntic* texts, there has always been a noticeable deviation. The ancient Indian astronomers suggested the well known corrections, besides the *manda* equation, which we call *evection* and *variation*.

The equation of centre (*manda* correction) was known in India even before Āryabhaṭa I (476 A.D.). In fact Āryabhaṭa himself gives the coefficient in the *manda* equation as $300'.25$. Brahmagupta in his *Uttara Khaṇḍa Khādyaka* gives the same as $301'.7$. However, it must be pointed out that out of the actual equation of centre, a part of it is combined with the second correction ("evection") and the combined equation is even in later *siddhāntic* texts.

In fact, this combined equation for the Moon was first given, among the Indian astronomers, by Mañjula (or Muñjula , 932 *A.D.*) in his *Laghumānasam* . P.C. Sengupta points out, "In form the equation is most perfect, it is far superior to Ptolemy's; it is above all praise." While the credit of discovering the Moon's second equation, among the Hindu astronomers, undoubtedly goes to Mañjula , it was Bhāskara II (1114 A.D.) who introduced it into his *siddhānta* .

The third equation for the Moon's position, "variation" was introduced in Indian astronomy by II in1150 A.D., four centuries before Tycho Brahe discovered it in the west.

The honour of introducing the fourth equation, to the Moon's position, now called "*annual equation*" goes to the highly dedicated astute astronomer from Orissa **Mm. Samanta Chandra Sekhar Simha** of the 19th century. He called it "*Digams′a*" *samskāra* and incorporated it in his remarkable text, *Siddhānta darpaṇa* . The constant coefficient in Chandra Sekhar's equation is $11'26.6''$. It is important to note that Tycho Brahe had given the coefficient as $4'30''$. The modern value is $11'10''$. Thus, Chandra Sekhar *Sāmanta's* value is far closer to the modern value. This accuracy of his value is truly remarkable in the light of the fact that the *Sāmanta* was trained exclusively in the orthodox Sanskrit tradition and totally ignorant of the English education or the western development of astronomy.

The modern expressions for the three above-said equations of the Moon are as follows:

1. Evection = $4586''\sin(2D-g)$ where $D = M - S$, the mean elongation of the moon (from the sun), *M* and *S* being the mean longitudes of the moon and the sun respectively and *g* is the mean anomaly of the moon (from its *perigee*).

In the context of Indian astronomy, the mean anomaly (*manda kendra*) is measured from the apogee (*mandocca*). If the perigee and the apogee of the moon are denoted respectively by *P* and *A*, then we have

Mean anomaly,

$$g = M - P = M - (A + 180^0)$$
$$= (M - A) - 180^0$$

so that the evection equation becomes

$$Evection = 4586''\sin\left[2D - \left(A + 180^0\right)\right]$$
$$= -4586''\sin\left[2D - (M - A)\right]$$

However, as defined in the $S\bar{u}rya\ siddh\bar{a}nta$,

Manda anomaly = *Mandocca* – Mean longitude = $A - M$

In which case

$$\text{Evection} = -4586''\sin[2D + MA]$$

where $MA = A - M$, the *manda anomaly* of the moon. In terms of the mean longitudes of the sun (*S*) and the moon (*M*) the *mandocca A*,we have

$$\text{Evection} = -4586''\sin(M - 2S + A) \qquad \dots (10.1)$$

2. Variation

$$\text{Variation} = 2370''\sin(2D) \qquad \dots (10.2)$$

where $A = M - S$, the moon's elongation from the sun. Samanta Chandra Sekhar in his $Siddh\bar{a}nta\ Darpa\d{n}a$ has taken this equation as

$$\frac{[R\sin 2[M - S]]}{90} \text{ or } 38'12''\sin 2D$$

$i.e.\ 2292''\sin(2D)$ where $R = 3438'$

3. Annual equation = $-668''\sin(g')$ where g' is the sun's mean anomaly

$i.e.\ g' = S - P'[P' : \text{Sun's perigee}]$

Here also, considering the sun's anomaly measured from his apogee (*mandocca*) A', as is the case in the $Siddh\bar{a}ntas$, we have

$$\text{Annual equation} = -668''\sin\left[S - (A' + 180^0)\right] = 668''\sin[S - A']$$

$$\text{Annual equation} = -668''\sin[A' - S] \qquad \dots (10.3)$$

where $\left[A'-S\right]$ is the sun's *manda kendra* (as defined in the $S\bar{u}rya\ \ siddh\bar{a}nta$).

Remark: Infact, the moon's annual equation happens to be a fraction of the sun's *mandaphala* (equation of centre). According to the modern values of the concerned coefficients we have

$$\text{Sun's equation of centre} = 6910''\sin\left(g'\right)$$

$$\text{Annual equation of the moon} = -668''\sin\left(g'\right)$$

$$\text{The ratio of the latter to the former} = \frac{-668}{6910} \approx -\frac{1}{10.34}$$

Samanta Chandra Sekhar has approximated this ratio to (-1/10) and taken Moon's annual equation = $\frac{-1}{10}$ Sun's *manda* equation .

As a respectful tribute to the Samanta Chandra Sekhar we shall continue to use the names $Tu\dot{n}g\bar{a}ntara$, $P\bar{a}k\d{s}ika$ and $Digam\acute{s}a$, given by him, respectively for evection, variation and annual equation.

Thus from (10.1), (10.2) and (10.3) we have three equation of the moon, besides the usual *mandaphala*, given by

a. $Tu\dot{n}g\bar{a}ntara\ \ Samsk\bar{a}ra$ (Evection) = $-4586''\sin\left(M-2S+A\right)$

b. $P\bar{a}k\d{s}ika\ \ Samsk\bar{a}ra$ (Variation) = $2370''\sin 2(M-S)$

c. $Digam\acute{s}a$ (Annual equation)

$$= -668''\sin\left(MK\right) \approx -\frac{1}{10.34}\ (Sun's\, equation)$$

where *MK* is the sun's *manda kendra* (anomaly of apsis) defined by $MK = A' - S(A' : \text{Sun's}\, mandocca)$.

Note: In the $Tu\dot{n}g\bar{a}ntara$ equation, given by Samanta Chandra Sekhar a part of the (modern) equation of centre is combined with evection equation. In the earlier $siddh\bar{a}ntic$ texts also, the *manda* equation included only a major part of the (modern) equation. However, in the proposed equations we are suggesting in this work the three equations (a) to (c) above correspond to the three equations adopted in modern astronomy.

11. The case of Budha and S′ukra

The traditional Indian astronomical text have always treated Budha and differently from the remaining viz., Kuja, Guru and in the context of determining their true positions.

While the mean position of the superior planets are taken as they are, in the case of Budha and S′ukra (the inferior planets) two special points called Budha $s'\bar{\imath}ghrocca$ and S′ukra $s'\bar{\imath}ghrocca$ are considered.

The position of the mean Ravi is itself taken as the position of both mean Budha and mean S′ukra . Again, while working out the $s'\bar{\imath}ghra$ equation, the argument of the relevant sine function is taken, for example in the case of Budha, as (*B* - *R*) as per the $S\bar{u}rya\ \ siddh\bar{a}nta$ convention, where *B* and *R* are respectively the Budha $s'\bar{\imath}ghrocca$ and the mean Ravi. In the case of superior planets, the order of the terms in the argument is reversed.

For example, for Guru, the argument in the sine term of the *s′īghra* equation is (*R* - *G*) where *G* is the mean position of Guru. For the superior planets the mean Sun (Ravi) is considered as their *s′īghrocca* . Nīlakaṇṭha somayāji (1444-1545 A.D.) points out in his *Tantra saṅgraha* that it is incorrect to have a differential treatment to the inferior and superior planets and that the sun is the common centre for the *s′īghra* equation to all the planets. This is truly a remarkable breakthrough in the history of mathematical astronomy in general and in Indian astronomy in particular (see Ramasubramanian et.al.,*Current Science*, May 1994). In fact, Nīlakantha's innovation, prompted by his *paramaguru* (1380-1460 A.D.), is highly suggestive of a heliocentric model of planetary motion, much before Copernicus. It is interesting that Samanta Chandra Sekhar though independently, proposed a similar model (see *Siddhānta Darpaṇa* , V, 6 and 7).

12. **Samanta on the Venus Transit**

In his *Siddhānta – darpaṇa*, Samanta Chandrasekhar Simha, mentioning his observation of the Venus transit of **1874 December 9**, says:

"To find on eclipse of the sun due to Venus, their *bimba* and the size of the *tārāgraha* is stated. In the *Kali* year 4975 (i.e.1874 A.D.) there was a solar eclipse due to *S′ukra* in the Vṛṣcika rās′i. Then the S′ukra *bimba* was seen as 1/32 of the Ravi *bimba* which is equal to 650 *yojanas*. Thus it is well proved that the *bimba* of *S′ukra* and planets is much smaller than that of the sun."

dṛṣṭaṃ s′ukrasya gāḍhāstamayajam
maṇḍalam caṇḍabhānau
kīṭāṃs′e pañcaviṃs′e gatavati
kalito'rthādrigo'bdhyabda vṛnde /
.....

The above translation is by Shri Arun Kumar Upadhyāya. The translator refers to the transit of Venus as 'eclipse of the sun due to Venus' (in fact, it is !).

However, Samanta himself, in his Sanskrit text calls the transit 'S′ukra's *gāḍhāstamaya*" i.e. **close** heliacal setting of Venus. It is interesting to note that Chintamani Raghunatha chary uses the same name, *gāḍhāsta* for the transit.

Siddhāntadarpaṇaḥ, XI, 110

Conclusion :

In this paper we have presented a few sample issues related to Samanta Chandra Sekhar's astronomical innovation from the point of view of computational veracity. We have seen how the two main items required for computing the planetary positions, namely epochal positions and the mean rates of motion (through the *bhagaṇas* in a kalpa*)* are fairly in order. The error in the rates of the slow moving *mandoccas* (apogees) hardly contributes significantly.

As the operational aspect of the Samanta's procedure for computing lunar eclipse, we have presented the total lunar eclipse of the current decade and seen its efficacy in predicting the circumstances of the eclipse within a permissible difference of a couple of minutes.

His contribution to the moon's equations is remarkable. Further we have brought out the significance of Samanta's observation of the 1874 Venus transit and his conclusion therefrom.

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Gaṇeśa Daivajña

– THE INDIAN ASTRONOMER WHO SIMPLIFIED THE ASTRONOMICAL PROCEDURES

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There is a general belief that after Bhâskara II there was a decline in the development of mathematics and astronomy in India. While there may be some truth in this belief, thanks to historical reasons, it is also true that the post–Bhāskara period saw an intensely creative activity in mathematics in the regions south of the Vindhyas. In fact Kerala became the cradle of tremendous and rich mathematical output – often anticipating developments in the European mathematics. In other parts of India too there shone great luminaries, like Gaṇēśa Daivajña, upholding the great tradition of Indian astronomy and mathematics.

In fact, the astronomical works of no other astronomer are in use among the makers of traditional Pañcāngās (astronomical almanacs) in most parts of India today as much as those of the great and popular astronomer, Gaṇēśa Daivajña.

1.1 Date and Place of Gaṇeœa and Keœava Daivajña

Gaṇēśa Daivajña's father was the famous astronomer Keœava Daivajña and his mother's name was Lakṣmi. He was born in 1507 AD (*śaka* 1429) at a place called Nandigrâma on the western sea-coast.

S.B.Dikshit points out that Nandigrâma is at present a village called Nandagaon in the Janjeera State in the Konkan region. It lies about 40 miles to the south of Mumbai (Bombay). The family belonged to the *gotra* (patrilineal ancestry) of *Kuśika*. Gaṇēśa's grand-father (Keœava's father) was Kamalâkara, also an eminent astronomer. Gaṇēśa's teacher in astronomy was his father himself while Keœava's *guru* was Vaijanâtha.

Gaṇēśa's father, Keśava, composed several works and commentaries on astronomy and astrology among which his astronomical work. *Grahakautuka* was highly respected. In fact, Keśava is regarded as one of the best observational astronomers of ancient and medieval India. In his Mitākṣārā auto-commentary on the *Grahakautuka*, Keœava observes:

"The figures as calculated from the *Brahmâ*, Āryabhaṭa and *Saura siddhāntas* exhibit a vast difference in the position of Mercury (Budha) and Venus (śukra). Saturn (śani) has shown an excess of five degrees when actually observed in the sky at the time of conjunction with stars and planets and while setting and rising… Similarly, a difference is recorded in epochal positions and in the annual rates of motion…".

"Hence the future calculators should calculate planetary positions by adopting the figures of revolutions increased or decreased in conformity with the actual observed phenomena of conjunctions, rising and setting of stars and planets in their own times. The writer (Keśava) has accordingly found out the mean position of the Moon, instead of its maximum equation of centre, by reversed steps, from the observation of the lunar eclipse at the ending moment of the fullmoon, since the equation of centre is neither positive nor negative. The Moon's apogee was finally fixed by

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reversing the steps of calculation, after observing the eclipse at the moment of the fullmoon in the celestial globe of observation since the maximum correction is neither additive nor subtractive. The moon's position was found to be 5 minutes (of arc) less as compared to that calculated from the *Sûryasiddhānta*. The apogee agreed with that of the Brahmapakṣa. Thus, the writer has calculated the positions of planets by a short method after observing their actual positions at the present time."

1.2 Works of Gaṇēśa Daivajña

Gaṇēśa Daivajña composed several important works on astronomy among which his astronomical treatise *Grahalāghavam* is the most famous. In fact, the remarkable popularity of the *Grahalāghavam* surpassed that of his father's *Grahakautuka* which was truly an important text in its own right.

Gaṇēśa's other works are: *Laghu- and* Bṛhat - Tithi Cintāmaṇi, a commentary of Bhâskara's *Siddhânta* Śirōmaṇi, a commentary on Bhâskara's *Lîlâlavamî* (called *Buddhivilāsinî*). *Vivâha vṛndāvanaṭīkā*, *Muhûrta tattvatîkâ*, *Śrâddha Nirṇaya* etc. Gaṇēśa himself mentions another work of his, *Parvanirṇaya* .

Among Gaṇēśa's works, the *Gragalâghavam* appears to have been composed first, believed to be when he was just 13 years old. The epoch of the *Gragalāghavam* is March 19, 1520 A.D. Monday at the mean sunrise at Ujjayini. The text consists of 187 *œlokas* distributed in 14 chapters. However, commentaries of Mallāri and Viúvanâtha contain a 15th chapter consisting of 15 *ślokâs* called Pañcānga Candragrahaṇam

The work, *Laghu tithi cintāmaṇi* was composed in *śaka* 1447 (1525 A.D.) and the *Buddhivilâsinî* commentary on Bhâskara's *Lîlāvatî* in the year 1545 A.D. Another work, *Pâta sāraṇi*, was composed some time after 1538 A.D.

In the *Grahalâghavam*, the positions of planets have been given for the moment of sunrise of Monday, the newmoon day of *Phālguṇa* of *śaka* 1441 corresponding to March 19, 1520 A.D. (Julian).

2. Popularity of *Gragalâghavam* and its commentators

As mentioned earlier, the *Grahalāghavam* of *GaGeœa Daivajña* is the most popular astronomical text, among the ancient and medieval texts, currently used in most parts of India. Further, among the *karaṇa* works (hand-works) on Indian astronomy, the *Grahalāghavam* is considered as the most comprehensive, exhaustive and easy to use text.

The *Grahalâghavam* carries with it very useful and authoritative commentaries by reputed astronomers like Gangâdhara (1586 A.D.), Mallâri (1602 A.D.) and Viśvanâtha (around 1612 A.D.)

Gangādhara's commentary on the *Grahalâghavam* is called *Manoramâ*. His father was Nârâya*ṇa* who authored *Muhûrta mārtāṇḍa*. Gangādhara was a Vâjasaneyî Brâhmin belonging to the *Kauœikagotra*, the same as that of Gaṇēśa Daivajña. Gangādhara lived in a village called Tapar, lying to the north of Ghṛṣṇeśvara(Lord Śiva) temple which is to the north of Devagiri (Daulatabad).

Viśvanātha was brother of the highly accomplished astronomer Viṣṇu who composed a Karaṇa with the epochal year 1608 A.D. This Karaṇa text is based on the *Sûrya siddhânta*. Viṣṇu also wrote a commentary on Gaṇēśa Daivajña's Bṛhat - Tithi Cintāmaṇi in which he explains the theory also. Viúvanâtha has written an *udâharaṇa* on his brother's Karaṇa.

The two popular commentators of *Grahalâghavam* are Viśvanātha and Mallâri. They were born in illustrious Mahārāṣṭrian Brāhmin families of astronomers. Mallâri's father was Divâkara, a pupil of Gaṇēśa. Kamalākara, author of the *Siddhānta tattva viveka* and Ranganâtha who wrote a commentary on the *Sûryasiddhānta* were descendants of Viśvanâtha and Mallâri. Nṛsimha, nephew and pupil of Gaṇēśa, wrote his commentary Harṣakaumudī in 1548. The other commentators are Gangâdhara (1586), Nārâyāṇ a (Kaśî, before 1635) and Kamalākara (before 1662).

Viśvanātha has given a large number of examples in his commentary to illustrate the methods of the *Grahalâghavam*. The *Grahalâghavam* is extensively used by the Pañcāng*a*-makers particularly in Maharashtra, Gujarat, northern parts of Karnataka, the Hyderabad Deccan region of Andhra Pradesh and by the Deccanis of Varanasi, Gwalior and Indore. It is pointed out that even the government almanacs published at Indore and Gwalior used the *Grahalâghavam* and the Tithicintāmaṇi of Gaṇēśa Daivajña.

3. Special features of *Grahalâghavam*

(i) Cakra (cycle) and Ahargana

Gaṇēśa has simplified the method of computations of the positions of planets which is otherwise laborious by the traditional method.

To avoid a huge number for the ahargaṇa (number of civil days since the epoch), Gaṇēśa has adopted an ahargaṇa cycle of 4016 days, approximately constituting 11 solar years. Therefore, the modified ahargaṇa being the remainder exceeding a completed number of cycles (of 4016 days each), never exceeds 4016 days and is hence handy.

From the point of view of a pañcānga-maker or a beginner who is ignorant of trigonometry, *Gragalâghavam* is easy to use since Gaṇēśa has completely dispensed with the trigonometric functions.

In fact, the dropping of trigonometric ratios has by no means seriously affected the accuracy of results. Very justifiably, Ganeœa has replaced the *sine* of an angle by a powerful algebraic approximation (due to Bhaskara I of 7^{th} century). Even the other Karaṇa texts, using the sine table, generally give the values of sines of angles only in intervals of 15° or $3\frac{3}{4}^\circ$. Therefore, their sine-values for intermediate angles, by linear interpolation, are only approximate.

(ii) Phase of the Moon

From the newmoon to the fullmoon, the *phase* of the Moon increases, given by

$$\text{phase} = \frac{(1+\cos d)}{2}$$

where *d* is the elongation *SME* of the earth (E) from the Sun (S) as seen from the Moon (M). In fact, we have

$d = 180° - E$, so that

$$\text{phase} = \frac{(1-\cos E)}{2}$$

where *E* is the elongation of the Moon from the Sun as seen from the earth. On a new moon day, $E = 0°$ so that the phase of the Moon is zero. On a fullmoon day, $E = 180°$ so that the phase of the Moon is one.

In Indian astronomy, the phase of the Moon is measured by the width of the illuminated part of the Moon which is called *sita* (or *œukla*). The width of the unilluminated part, equal to the difference between the Moon's diameter and the *sita* is called *asita*. In fact,

$$sita = \frac{(M-S)\times(\text{Moon's ang. diameter})}{180}$$

where *M* and *S* denote the celestial longitudes of the Moon and the Sun respectively in degrees.

Gaṇēśa Daivajña gives the formula $sita = \left(1-\frac{1}{5}\right)T$ aEgulas.

where *T* is the number of *tithis* elapsed in the bright fortnight (*œukla pakca*) and the Moon's diameter is taken as 12 *angulas*. Of course, this formula is approximate and a similar formula is given by Brahmagupta.

(iii) Rationale for four sides to form trapezium

Bhâskara II investigates the possibility of four sides forming a trapezium. He gives the condition: "in a trapezium the sum of the other flank side and the face is smaller than the sum of the smaller flank side and the base." (*Lîlâvatî*, 185)

Gaṇēśa Daivajña has provided a rationale for this statement of Bhâskara II.

(iv) Proof of the Śulva theorem (Pythagoras' theorem)

Gaṇēśa Daivajña in his *Buddhivilâsinî* commentary on the *Lîlâvatî* provides of fully geometrical proof for the geometric-algebraic rationale provided by Bhâskara II for the so-called Pythagoras theorem on a right-angled triangle.

Let *ABC* be a triangle right-angled at *A* (see Fig). Let *AD* be the perpendicular to *BC*. Then the triangles *ADB*, *CDA* and *CAB* (the vertices are in the order of correspondence of equal angles) are *similar*. Therefore, from triangles *ADB* and *CAB*, we have

$$\frac{AB}{BC} = \frac{BD}{AB} \text{ or } BD = \frac{AB^2}{BC} \quad \ldots(1)$$

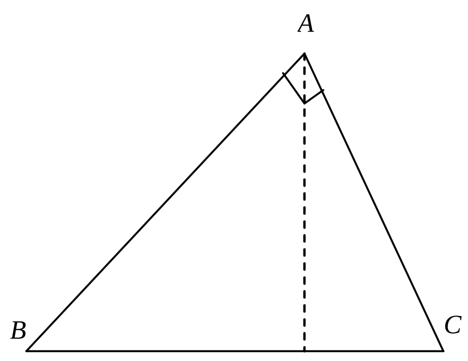
Similarly, from triangles *CDA* and *CAB*,

$$DC = \frac{AC^2}{BC} \quad \ldots(2)$$

From (1) and (2), we get

$$BD + DC = BC = \frac{AB^2 + AC^2}{BC} \text{ or}$$

$$BC^2 = AB^2 + AC^2$$



Proof of the Śulva theorem

(v) Construction of rational quadrilaterals

Gaṇēśa Daivajña, commenting on the method of Brahmagupta to obtain a quadrilateral with rational sides, says that four rational right-angled triangles (*jâtyas*) are to be formed out of two basic rational right triangles as follows:

If $m^2 - n^2,\ 2mn,\ m^2 + n^2$ and $p^2 - q^2,\ 2pq,\ p^2 + q^2$

are the sides of two rational right triangles, then according to GaGeœa, the following are the triangles out of which the quadrilaterals are built up:

(i) $(m^2 - n^2)(p^2 - q^2),\ 2mn(p^2 - q^2),\ (p^2 - q^2)(m^2 + n^2)$

(ii) $(m^2 - n^2)(2pq),\ 4mnpq,\ 2pq(m^2 + n^2)$

(iii) $(p^2 - q^2)(m^2 - n^2),\ 2pq(m^2 - n^2),\ (p^2 + q^2)(m^2 - n^2)$

(iv) $(p^2 - q^2)2mn,\ 4pqmn,\ (p^2 + q^2)(2mn)$

However, T.A.Sarasvati Amma points outs that this is a further improved procedure by Gaṇēśa.

(vi) Evaluation of π

Āryabhaṭa (b. 476 A.D.) has given the value of π as

and he points out that this value is approximate (*âsanna*).

Bhâskara II also gives the value

$$\pi = \frac{3927}{1250} = 3.1416 \qquad \text{and} \qquad \pi = \frac{355}{113} = 3.1415929$$

the former value being the same as the one given by Âryabhama (obtained by removing the common factor 16 in the ratio).

The mode of arriving at this value of Π is by considering the perimeter of a regular polygon inscribed in a circle. The ratio of the perimeter of the polygon to the diameter of the circle approximates the constant . Of course, the more the sides of the polygon, the better is the approximation.

Gaṇēśa Daivajña suggests that the number of sides of the inscribed polygon, starting from 12, is successively doubled as 24, 48 until it is 384. The diameter of the circle is taken as 100 units. Then the ratio of the perimeter of the 384 sided polygon inscribed in a circle of diameter 100 would give the approximate value of π as 3927/1250.

Gaṇēśa Daivajña's commentary, *Buddhivilâsinî* on the *Lîlâvatî* is an extremely useful text to understand the rationales for the formulae and methods used by Bhâskara II and his predecessors.

4. Some important approximations in the Grahalâghavam

(1) *Mandaphala* of *Ravi*

The usual formula, according to the traditional texts:

Mandaphala of Ravi $\quad = \frac{a}{r} \sin m$

where m = *manda* anomaly of the sun, a = 14^0, *paridhi* of the sun's epicycle and $R = 360^0$ *paridhi* of Kakṣa Vṛtta

Ganeśa Daivajna's formula:

Mandaphala of the sun = $$\frac{\left[20-\frac{BMK}{9}\right]\frac{BMK}{9}}{57-\left[\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right]\Big/9}$$

where BMK = *Bhuja of Mandakendra*. This formula is derivable from śrīpati Bhamma's (1039 AD)expression:

dōḥ kōṭibhāgārahitābhihitāḥ khanāgacandrāstadīyacaraṇōnaśarārkadigbhiḥ

te vyāsakhaṇḍaguṇitā vihṛtāḥ phalantu jyābhirvināpi bhanatō bhurakōṭijīve||

i.e., *Mandakendra jyā* = $$\frac{(180-MK)MK\times 120}{10125-\frac{(180-MK)}{4}MK}$$

$$=\frac{\left(20-\frac{MK}{9}\right)\frac{MK}{9}\times 480}{500-\left(20-\frac{MK}{9}\right)\frac{MK}{9}}$$

(taking the l.c.m and multiplying Nr. & Dr. 9 x 9)

Now, the *parama manda phalam* of the sun $=\frac{125}{57}$

$\therefore$ *Mandaphala* of the sun $=\frac{125}{57}\times\frac{MK\ jya}{120}$

This results in the above formula.

Let $\left(\frac{a}{R}\right)\frac{180}{\pi}=\frac{125}{57}$

$\therefore a=\frac{2\pi\times 125}{57}$ (taking $R=360$)

$\equiv 13^\circ.7789115$ (taking $\pi = 3.1416$)

Note: Based on the modern value of the eccentricity of the earth's orbit, *a* lies between $11^\circ.8078$ $12^\circ.3128$

(2) *Gatiphalam* of the sun (True Motion of the Sun)

Ravi gati phalam $=\frac{\left(11-\frac{koti}{20}\right)\frac{koti}{20}}{13}$ . . . (1)

Consider the ratio of *trijyas* of the *B[had-jyā* and *Laghujyā*

$$=\frac{3438}{120}=\frac{382\times 9}{120}\approx\frac{13\times 9}{2\times 2}$$

$$=\left(\frac{22-9}{2}\right)\left(\frac{9}{2}\right)=\left(11-\frac{9}{2}\right)\frac{9}{2}$$

$$=\left(11-\frac{90}{20}\right)\frac{90}{20}$$

Now, *paramakoti* = $90°$; we get taking *koti* = $90°$:

the *parama gati phalam* = $\frac{9}{4}$ from (1).

$$\therefore \textit{Ravi gati phalam} = \frac{\left[\left(11-\frac{koti}{20}\right)\frac{koti}{20}\right]\times\frac{9}{4}}{13\times\frac{9}{4}}=\frac{\left[\left(11-\frac{koti}{20}\right)\frac{koti}{20}\right]}{13}$$

Example: For *MK* = $43°46'18''$, *koti* $= 90° - (43°46'18'')$

i.e., $koti = 46°13'42''$.

$\therefore$ *Ravi gati phalam*, $\Delta n = 1'32''41'''$ (-ve) from (1)

Modern formula, $\Delta n = -\frac{b}{R}\cos M\left(\frac{\Delta M}{\Delta t}\right)$

$$=-\frac{14}{360}\cos(43°46'18'')(59'08'') = -1'39''54'''.$$

Comparing the *Grahalaghavam* value for the *Ravi gati phalam* (Δn) ith the one obtained from the trigonometry-based formula, we observe that the difference is just about $7''$.

(3) *Gatiphalam* [i.e. True Daily Motion] of the Moon

$$\textit{Candra gati phalam} = \left[\left(11-\frac{koti}{20}\right)\frac{koti}{20}\right]\left(2+\frac{2}{6}\right) \quad \ldots(2)$$

Consider the ratio of *trijyas* of the *Bṛhad-jyā* and *Laghujyā*

$$=\frac{3438}{120}=\frac{382\times 9}{120}\cong\frac{13\times 9}{2\times 2}$$

$$=\left(\frac{22-9}{2}\right)\frac{9}{2}=\left(11-\frac{9}{2}\right)\frac{9}{2}=\left(11-\frac{90}{20}\right)\left(\frac{90}{20}\right)$$

Now, *paramakoti* = $90°$; we get the *paramagatiphalam* $=\frac{273}{4}$ from (1).

By proportion,

$$\textit{Candra gati phalam} = \frac{\left[\left(11-\frac{koti}{20}\right)\frac{koti}{20}\right]\frac{273}{4}}{13\times\frac{9}{4}}$$

$$=\left[\left(11-\frac{koti}{20}\right)\frac{koti}{20}\right]\left(2+\frac{2}{6}\right)$$

Example: Candra's *Mandakedra* = $3^R 25°12'17'' \equiv M$

(*Manda* anomaly) *MK* = $115°12'17''$

$\therefore$ *Bhuja* of *MK* $= (180° - MK) = 64°47'43''$

Koti of *MK* = $90°$ – *Bhuja* = $25°12'17''$

(i) *Candra gati phalam* (from *GL* formula) = $28'38''2'''$

(ii) Modern formula: $\Delta n = -\frac{b}{R}\cos M\left[\frac{dM}{dt} - \frac{dA}{dt}\right] = 28'44''46'''$

where $b = 31°,\ R = 360°$

In the case of the moon, the difference between the values of Δn (i.e., the correction to the mean daily motion to get the true one) according to *GL* and the trigonometry-based expression is just about $7''$ which is negligible.

5. Lunar eclipse computation

In ancient and medieval astronomical texts (*siddhāntas, tantras* and *karaṇas*) great importance is given to the phenomenon and computation of eclipses (grahaṇa, *uparāga*). The Indian astronomers used to put to test their theories and computations in respect of positions of the heavenly bodies – especially the sun and the moon –on the occasions of the eclipses. As and when disagreements occurred between the observed and the computed positions, the great savants of Indian astronomy revised their parameters and when necessary even the computational procedures. Improving the computations of eclipses – based on sustained observations over long periods of time – was an important target of *siddhāntic* astronomers.

While the geometrical configurations and mathematical procedures for computations of eclipses presented by the traditional Indian astronomers are quite sound, the relevant parameters, over a period of centuries, need to be upgraded periodically.

Despite the approximations introduced in *GL*, doing away with trigonometric ratios, the predictions of eclipses are fairly reliable on account of improved and updated values of the related parameters.

The procedure, as described in *GL*, is presented briefly in what follows. Based on Gaṇēśa's procedure we have written a computer program and the same is demonstrated with an example, of a lunar eclipse of his time.

(i) Sun's angular diameter (*ravi bimbam*),

SDIA = (SDM – 55)/5 + 10 *angulas*

where SDM is the sun's true daily motion in minutes of arc (*kalās*).

(ii) Moon's angular diameter (*candra bimbam*)

MDIA = MDM/74 *angulas*

where MDM is the moon's true daily motion in minutes of arc (*kalās*).

(iii) Angular diameter of the earth's shadow (*bhūcchāyā bimbam*).

SHDIA = [3(MDIA)/11+3(MDIA)-8] *aEgulas*

Note: 1 *aEgula* = 3 *kalās* a" $3'$

(iv) Latitude of the moon (*candra śara*)

MLAT (*śara*) = 11(M – R)/7

where M and R respectively the true longitudes of the Moon and Rāhu (the ascending node of the moon) and (M – R) considered here is the *bhuja* (not exceeding $90°$) of the difference.

Note: If (M – R), is in III quad. then the argument,

bhuja = (M – R) – $180°$;

If (M – R) is in IV quadrant, the *bhuja* = $360°$ – (M – R). On the other hand, if (M – R) is in II quadrant, the *bhuja* = $180°$ – (M – R).

Remark: The approximate formula follows from

$$120 \sin\theta \approx 72\theta/35 \qquad \ldots(1)$$

when θ is small, as given by GaGeśa (see *GL, Praśnādhikāra*, 22).

According to the traditional texts the moon's latitude (*śara*) is given by

$$\beta = 270' \sin(M-R) = 270 \times \frac{72}{120 \times 35}(M-R) \ \textit{kalās}.$$

$$\beta = \frac{162}{35}(M-R) \ \textit{kalās}.$$

Dividing the above result by 3, we get

$$\beta = \frac{54}{35}(M-R) \approx \frac{11}{7}(M-R) \ \textit{angulas}.$$

The approximations in this case are justified since under the possible circumstances of an eclipse the argument (M – R) is indeed small.

(v) The amount of obscured portion of the full-moon,

$$\textit{Grāsa} = \frac{1}{2}[\textit{chādaka} \text{ dia.} + \textit{chādya} \text{ dia}] - \text{śara}$$

where *chādaka* and *chādya* are the eclipsing and the eclipsed bodies. The formula is common to both the lunar and solar eclipses. In the case of a lunar eclipse, the moon is the *chādya* and the earth's shadow is *chādaka*.

(vi) $$\textit{Mānaikya khaṇḍa} = \frac{1}{2}[\textit{chādaka} \text{ dia.} + \textit{chādya} \text{ dia.}]$$

so that we have *Grāsa* = *Mānaikya khaṇḍa* – *śara*

Therefore,

(a) if *Mānaikya khaṇḍa* < *śara* (i.e. *Grāsa* < 0), there will be **no eclipse**.

(b) If *Grāsa* > *Chādya* dia, then the **eclipse is total** (*khagrāsa grahaṇa*).

In that case, *Khagrāsa* = *Grāsa* – *Chādya* diameter

(vii) Half duration of the eclipse and totality:

(a) In Gaṇēśa simplified procedure,

Let $$x = \sqrt{\left[\frac{1}{2}(\text{SHDIA} + \text{MDIA}) + \acute{s}ara\right] \times 10 \times gr\bar{a}sa}$$

Then, the half-duration of the eclipse,

HDUR = $(x - x/6)$ /MDIA = 5x / 6 (MDIA) *gh.*

(where *gh.* = *ghamikās;* 60 *ghamikās* = 1 day; 1*gh* = 24 minutes).

(b) Similarly

$$\text{let } x = \sqrt{\left[\frac{1}{2}(\text{SHDIA} + \text{MDIA}) + \acute{s}ara\right] \times 10 \times khagr\bar{a}sa}$$

Then, the **half-duration of totality**,

$$\text{THDUR} = [y - y/6]/\text{MDIA} = (5y/6)/(\text{MDIA})\, gh.$$

(viii) First and second halves of eclipses and totality:

The difference (true Sun – Rāhu), called *vyagu*, at the instant of opposition is considered and its *bhuja* is determined. The product $2 \times$ *bhuja* in degrees is put in two places as *palas* (i.e. *vighṭīs*).

(1) If the *vyagu* is in an even quadrant then (2 x *bhuja*) in *palas* is subtracted from and added to the *madhya sthiti* (i.e., mean half-duration HDUR in *gh.* obtained earlier), respectively, to get the corrected first and second half-durations (called *sparśa sthiti* and *mokṣa sthiti*).

(2) If the *vyagu* is in an odd quadrant, then 2 x (*bhuja*) in *palas* is added to and subtracted from the *madhya sthiti* in *gh*., respectively, to get the corrected *sparśa* and *mokṣa* half-durations.

Similar operations are carried out to get the first and the second half-durations of *totality* by considering the *marda* duration THDUR instead of the *sthiti*.

Example

The lunar eclipse of **May 2, 1520 A.D. (Julian),** Wednesday, is considered. This date falls in the epochal year of *GL* and is taken from the *Epigraphia Indica* (see Vol. p.237). The example is worked out using a computer program designed by us.

***Grahalāghavam* : Position of Sun, Moon and Rahu**

Date :	:	Year : 1520	Month : 5	Date : 2
Time (after sunrise)	:	Hours : 0	Mins : 0	
Name of Place	:	Ujjayini		
Longitude (-ve for west)	:	Deg. : 75	Min : 45	
Latitude (-ve for south)	:	Deg. : 23	Min : 11	
Week Day	:	Wednesday		
Cakras	:	0		
Ahargaṇa	:	44 Epoch : 19-3-1520 J]		
True Sun	:	$35°35'23''$		
True Moon	:	$205°56'7''$		
Rāhu	:	$25°18'4''$		

***Grahalāghavam* : Lunar Eclipse**

Sun's true daily motion (SDM)	:	$57'30''$
Moon's true daily motion (MDM)	:	$736'15''$
Time of opposition after midnight (LMT)	:	$24^h 21^m 37^s$

True Sun at Oppn.	:	35°19′22″
True Moon at Oppn.	:	215°19′22″
Node (Rāhu) at Oppn.	:	25°15′39″
Moon's diameter (in *angulas*)	:	9.949405
Shadow's diameter (in *angulas*)	:	24.56169

Eclipse is Possible

(Northern) *śara* (in *aEgulas*)	:	15.81192
Grāsa (in *aEgulas*)	:	1.443631

Lunar Eclipse Partial

Madhya stihiti (in *gh.*)	:	1.829996
Sparśa sthiti (in *gh.*)	:	2.165401
Mokca sthiti (in *gh.*)	:	1.494592

Summary of Lunar Eclipse **L.M.T.**

		h	*m*	*s*
Sparśa (Beginning) time	:	23	29	39
Madhya (Middle) of Eclipse	:	24	21	37
Mokca (Ending) Time	:	24	57	29

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ADDENDUM

Blank

Amaranthine flower in the math world sapling

(Srinivasa Ramanujan)

G.Suresh Babu*

Come on friends,
let us peep through the past for a while in TIME MACHINE ….
2010…2000…1980…1960…1940…1920….1900…1896
Asia…India….Tamilanadu…..Erode
It is a school in a remote village where mathematics class is going on. Let us observe few instances..

1. The teacher is explaining that when a number is divided by itself, the outcome is 1. does it make you to ask the question, "Is it true when the number is zero?" Not only did a lean 9 years old boy among the class raise it, but also found the answer.

Take for instance 45/5=9. This is true because 9x5=45. Now consider 0/0

since 0x any number is 0, 0/0 gives any number. In other words, 0/0 cannot give a unique answer as in divisions with non zero number divisor.

So 0/0 should be indeterminate.

This is what the 9 years boy did. He was confident enough to raise it and explain it in the class to the wonder of his classmates and his teacher. His teacher did not know it as it was not in the syllabus and the textbook and no such question had ever been asked at that level.

2. The teacher is teaching square and square root. You are able to understand that $9=3^2$ and $3=\sqrt{9}$. Does it stir your imagination and set you on a voyage of pattern finding?

See what that lean boy did . He writes

$$3=\sqrt{(1+8)}$$

$$3=\sqrt{(1+\{2x4\})}$$

$$3=\sqrt{(1+\{2x\sqrt{16}\}}\,)$$

$$3=\sqrt{(1+\{2x\sqrt{1+15}\}}\,)$$

$$3=\sqrt{(1+\{2x\sqrt{[1+3x5]}\}}\,)$$ and so on.

He stops when he seized of and hence thrilled by an emerging pattern.

1, 2x4, 3x5, 3, 4x6, 4, 5x7 and so on. At once he jots down a new mathematical clothing for 3.

$\mathbf{3}=\sqrt{1}\ \mathbf{+2}\sqrt{1}\ \mathbf{+3}\sqrt{1}\ \mathbf{+4}\sqrt{1}\ \mathbf{+5}\sqrt{1}$ + etc. He baffles others by rubbing 3 off and posing the problem: Evaluate the **"net of square roots."**

Next day…

3. The same maths teacher is demonstrating the repeated multiplication of the same number and the laws of exponents or indices. he is teaching them prime factorization involving use of exponents.
You learn to write
72 = 2 x 2 x 2 x 3 x 3 = 2^3 x 3^2
Does it make you look for and devise a pattern? See what that lean boy does:
First he takes (2^2) x (6 exp 6). From this he gets 3^3 x 3^3 x 4 exp 4 and writes
2^2 x 6 exp 6 = 3^3 x 3^3 x 4 exp 4
His aesthetic sense drives him to develop it to have a beautiful sequence.

(1 exp 1) x (1 exp 1) x (2 exp 2) x (6 exp 6) = (3 exp 3) x (3 exp 3) x (4 exp 4)

what is the beauty in this? Can you discover it and make your own examples? Find the sums of the bases on either side of the equality sign. Are they not equal? Next find the sums of the exponents on either side. Of course they are equal as the bases and their exponents on either side. Of course they are equal as the bases and their exponents are the same in each case. But the bases and their powers on one side of the equality are different from those on the other side and yet the products are equal.
He gets immense delight and finds himself reeling off many more such beautiful equalities and recording them in his notebooks which are today world famous.

4. the teacher is talking about prime numbers. A prime number has two and only two distinct factors, 1 and the number itself. This gives a test to find if a number is prime.
If a number has two and only two distinct divisors, the number is prime. All this you understand and feel comfortable about. Does it make you eager to play with prime numbers?
See, the lean boy takes sets of consecutive primes and tries to find an equality among them-an exercise in pure intuition. No formula can help you here. He succeeds and jots down his findings in his notebooks.
Taking 2, 3, 5 and 7, a four element set of consecutive prime numbers,
he writes 3 +7 = 2.5 (dot means times) Next he takes 2, 3, 5, 7 and 11, a five element set of consecutive prime numbers and discovers an equality among them. 2.5 + 11 = 3.7 and so on. Can you add your own?

Think… **Thin**k…. T**hink** …Th**ink…..1896..to…present**

What did you infer from this? The first thing is that the 9 year old lean boy is none other than **Srinivasa Ramanujan Iyengar** who popularly known as ***srinivasa ramanujan*** was the strangest man in all of mathematics, probably in the entire history of science. He has been compared to a bursting supernova, illuminating the darkest, most profound corners of mathematics.

Numerologically speaking, the name itself derives the magic number **9**
Which is sum of all alphabetical digits (A=1,B=2…Y=25,Z=26)
The number ***nine*** is a most remarkable number in many respects. It is held in great reverence by all who practice the occult sciences and spiritual fields .
Not only that, in mathematical science it possesses properties and powers which are found in no other number.

Spiritually speaking the total number of condos in the holy **Bhagvadgeetha** and the **Mahabharatha** is 18 (1+8=9)
The number of days that battle took place in **Mahabharatha is 18**
Lord Sri Rama was born on the **9th** day as per **Ramayana.**

the sum of the digits which form its multiples are themselves always a multiple of *nine*; e.g., 2 x 9 = 18 (and 1+8=9); 3 x 9= 27 (and 2+7=9); 4 x 9 = 36 (and 3+6=9); 5 x 9 = 45 (and 4+5=9), etc., etc.; and so with the larger numbers: 52843 x 9 = 475587 (and 4+7+5+5+8+7=36, and 3+6=9). (2)

The sum of its multiples through the nine digits = 405, or 9 times 45.
It is the *last* of the digits, and thus marks the *end*; and is **significant of the *conclusion* of a matter.**

It is akin to the number ***six,*** six being the sum of its factors (3x3=9, and 3+3=6), and is thus significant of the *end of man*, and the summation of all man's works. *Nine* is, therefore,
THE NUMBER OF FINALITY OR JUDGMENT,
for judgment is committed unto **Jesus** as "**the Son of man"** (John 5:27; Acts 17:31). It marks the completeness, the end and issue of all things as to man—the judgment of man and all his works.
It is a *factor* of 666, which is 9 times 74.
The gematria of the word **"Dan,"** which means a judge, is 54 (9x6).
"th orgh mou" (*tee orgee mou*), my wrath, = 999 (Heb 3:11).
The solemn **amhn** (*ameen*), *amen*, or "verily," of our Lord, amounts also to 99, summing up and ending His words. The sum of the 22 letters of the Hebrew alphabet is 4995 (5x999). It is stamped, therefore, with the numbers of ***grace*** and ***finality*****.**

Srinivasa Ramanujan Iyengar (best known as Srinivasa Ramanujan) was born on December 22, 1887,in Erode about 400 km from Chennai, formerly known as Madras where his mother's parents lived.
After one year he was brought to his father's town, Kumbakonam. His parents were K. Srinivasa Iyengar and Komalatammal. He passed his primary examination in 1897, scoring first in the district and then he joined the Town High School. In 1904 he entered Kumbakonam's Government College as F.A. student. He was awarded a scholarship. However, after school, Ramanujan's total concentration was focused on mathematics. The result was that his formal education did not continue for long. He first failed in Kumbakonam's Government College. He tried once again in Madras from Pachaiyappa's College but he failed again.

During schooling, he came across a book entitled **A Synopsis of Elementary Results in Pure and Applied Mathematics** by **George Shoobridge Carr**. The title of the book does not reflect its contents. It was a compilation of about 5000 equations in algebra, calculus, trigonometry and analytical geometry with abridged demonstrations of the propositions. Carr had compressed a huge mass of mathematics that was known in the late nineteenth century within two volumes. Ramanujan had the first one. It was certainly not a classic. But it had its positive features. According to Kanigel, "one strength of Carr's book was a movement, a flow to the formulas seemingly laid down one after another in artless profusion that gave the book a sly seductive logic of its own." This book had a great influence on Ramanujan's career. However, the book itself was not very great. Thus Hardy wrote about the book: "He (Carr) is now completely forgotten, even in his college, except in so far as Ramanujan kept his name alive". He further continued, "The book is not in any sense a great one, but Ramanujan made it famous and there is no doubt it influenced him (Ramanujan) profoundly". We do not know how exactly Carr's book influenced Ramanujan but it certainly gave him a direction. 'It had ignited a burst of fiercely single-minded intellectual activity'. Carr did not provide elaborate demonstration or step by step proofs. He simply gave some hints to proceed in the right way. Ramanujan took it upon himself to solve all the problems in Carr's Synopsis. And as **E. H. Neville**, an English mathematician, wrote : "In proving one formula, as he worked through Carr's synopsis, he discovered many others, and he began the practice of compiling a notebook." Between 1903 and 1914 he had three notebooks.

While Ramanujan made up his mind to pursue mathematics forgetting everything else but then he had to work under extreme hardship. He could not even buy enough paper to record the proofs of his results. Once he said to one of his friends, "when food is problem, how can I find money for paper? I may require four reams of paper every month." In fact Ramanujan was in a very precarious situation. He had lost his scholarship. He had failed in examination. What is more, he failed to prove a good tutor in the subject which he loved most. At this juncture, Ramanujan was helped by R. Ramachandra Rao, then Collector of Nellore.Ramchandra Rao was educated at

Madras Presidency College and had joined the Provincial Civil Service in 1890. He also served as Secretary of the Indian Mathematical Society and even contributed solution to problem posed in its Journal. The Indian Mathematical Society was founded by V.Ramaswami Iyer, a middle-level Government servant, in 1906. Its Journal put Ramanujan on the world's mathematical map. Ramaswami Iyer met Ramanujan sometime late in 1910. Ramaswami Iyer gave Ramanujan notes of introduction to his mathematical friends in Chennai (then Madras). One of them was P.V. Seshu Iyer, who earlier taught Ramanujan at the Government College. For a shortperiod (14 months) Ramanujan worked as clerk in the Madras Port Trust which he joined on March 1,1912. This job he got with the help of S. Narayana Iyer. Ramanujan's name will always be linked to **Godfrey Harold Hardy**, a British mathematician. It is not because Ramanujan worked with Hardy at Cambridge but it was Hardy who made it possible for Ramanujan to go to Cambridge. Hardy, widely recognised as the leading mathematician of his time, championed pure mathematics and had no interest in applied aspects. He discovered one of the fundamental results in population genetics which explains the properties of dominant, and recessive genes in large mixed population, but he regarded the work as unimportant.

Encouraged by his well-wishers, Ramanujan, then 25 years old and had no formal education, wrote a letter to Hardy on January 16, 1913. The letter ran into eleven pages and it was filled with theorems in divergent series. Ramanujan did not send proofs for his theorems. He requested Hardy for his advice and to help getting his results published. Ramanujan wrote : "I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £ 20 per annum. I have had no university education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at mathematics. I have not trodden through the conventional regular course which is followed in a university course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as "startling"… I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me ". The letter has become an important historical document**. In fact, 'this letter is one of the most important and exciting mathematical letters ever written'**. At the first glance Hardy was not impressed with the contents of the letter. So Hardy left it aside and got himself engaged in his daily routine work. But then he could not forget about it. In the evening Hardy again started examining the theorems sent by Ramanujan. He also requested his colleague and a distinguished mathematician, John Edensor Littlewood (1885-1977) to come and examine the theorems. After examining closely they realized the importance of Ramanujan's work. As C.P. Snow recounted, 'before mid-night they knew and knew for certain' **that the writer of the manuscripts was a man of genius'**. Everyone in Cambridge concerned with mathematics came to know about the letter.

Many of them thought 'at least another **Jacobi** in making had been found out'. **Bertrand Arthur William Russell** (1872-1970) wrote to Lady Ottoline Morell. "I found Hardy and Littlewood in a state of wild excitement because they believe, they have discovered a second Newton, a Hindu Clerk in Madras … He wrote to Hardy telling of some results he has got, which Hardy thinks quite wonderful." Fortunately for Ramanujan, Hardy realised that the letter was the work of a genius. In the next three months Ramanujan received another three letters from Hardy. However, in the beginning Hardy responded cautiously. He wrote on 8 February 1913. To quote from the letter. "I was exceedingly interested by your letter and by the theorems which you state. You will however understand that, before I can judge properly of the value of what you have done it is essential that I should see proofs of some of your assertions … I hope very much that you will send me as quickly as possible at any rate a few of your proofs, and follow this more at your leisure by more detailed account of your work on primer and divergent series. It seems to me quite likely that you have done a good deal of work worth publication; and if you can produce satisfactory demonstration I should be very glad to do what I can to secure it" . In the meantime Hardy started taking steps for bringing Ramanujan to England. He contacted the Indian Office in London to this effect. Ramanujan was awarded the **first research scholarship** by the Madras University. This was possible by the recommendation of Gilbert Walker, then Head of the Indian Meteorological Department in Simla. Gilbert was not a pure mathematician but he was a former Fellow and mathematical lecturer at Trinity College, Cambridge. Walker, who was prevailed upon by

Francis Spring to look through Ramanujan's notebooks wrote to the Registrar of the Madras University : "The character of the work that I saw impressed me as comparable in originality with that of a Mathematical Fellow in a Cambridge College; it appears to lack, however, as might be expected in the circumstances, the completeness and precision necessary before the universal validity of the results could be accepted. I have not specialized in the branches of pure mathematics at which he worked, and could not therefore form a reliable estimate of his abilities, which might be of an order to bring him a European reputation. But it was perfectly clear to me that the University would be justified in enabling S. Ramanujan for a few years at least to spend the whole of his time on mathematics without any anxiety as to his livelihood."

Ramanujan was not very eager to travel abroad. In fact he was quite apprehensive. However, many of his well-wishers prevailed upon him and finally Ramanujan left Madras by S.S. Navesa on March 17, 1914. Ramanujan reached Cambridge on April 18, 1914. When Ramanujan reached England he was fully abreast of the recent developments in his field. This was described by J. R. Newman in 1968: "Ramanujan arrived in England abreast and often ahead of contemporary mathematical knowledge. Thus, in a lone mighty sweep, he had succeeded in recreating in his field, through his own unaided powers, a rich half century of European mathematics. One may doubt whether so prodigious a feat had ever been accomplished in the history of thought." Today it is simply futile to speculate about what would have happened if Ramanujan had not come in contact with Hardy. It could happen either way. But then Hardy should be given due credit for recognizing Ramanujan's originality and helping him to carry out his work. Hardy himself was very clear about his role. "Ramanujan was", Hardy wrote, "my discovery. I did not invent him — like other great men, he invented himself — but I was the first really competent person who had the chance to see some of his work, and I can still remember with satisfaction that I could recognize at once what I treasure I had found."It may be noted that before writing to Hardy, Ramanujan had written to two well-known Cambridge mathematicians viz., H.F. Baker and E.W. Hobson. But both of them had expressed their inability to help Ramanujan.

Ramanujan was awarded the B.A. degree in March 1916 for his work on 'Highly composite Numbers' which was published as a paper in the Journal of the London Mathematical Society. He was the second Indian to become a Fellow of the Royal Society in 1918 and he became one of the youngest Fellows in the entire history of the Royal Society. He was elected "for his investigation in Elliptic Functions and the Theory of Numbers." On 13 October 1918 he was the first Indian to be elected a Fellow of Trinity College, Cambridge. Much of Ramanujan's mathematics comes under the heading of number theory — a purest realm of mathematics. The number theory is the abstract study of the structure of number systems and properties of positive integers. It includes various theorems about prime numbers (a prime number is an integer greater than one that has not integral factor). Number theory includes analytic number theory, originated by **Leonhard Euler** (1707-89); geometric theory - which uses such geometrical methods of analysis as Cartesian co-ordinates, vectors and matrices; and probabilistic number theory based on probability theory. What Ramanujan did will be fully understood by a very few. In this connection it is worthwhile to note what Hardy had to say of the work of pure mathematicians: "What we do may be small, but it has certain character of permanence and to have produced anything of the slightest permanent interest, whether it be a copy of verses or a geometrical theorem, is to have done something beyond the powers of the vast majority of men." In spite of abstract nature of his work Ramanujan is widely known.

Ramanujan was a mathematical genius in his own right on the basis of his work alone. He worked hard like any other great mathematician. He had no special, unexplained power. As Hardy, wrote: "I have often been asked whether Ramanujan had any special secret; whether his methods differed in kind from those of other mathematicians; whether there was anything really abnormal in his mode of thought. I cannot answer these questions with any confidence or conviction; but I do not believe it. My belief that all mathematicians **think**, at bottom, in the same kind of way, and that Ramanujan was no exception." Of course, as Hardy observed Ramanujan "combined a power of generalization, a feeling for form and a capacity for rapid modification of his hypotheses, that were often really startling, and made him, in his peculiar field, without a rival in his day.

Here we do not attempt to describe what Ramanujan achieved. But let us note what Hardy had to say about the importance of Ramanujan's work. "Opinions may differ as to the importance of Ramanujan's work, the kind of standard by which it should be judged and the influence which it is likely to have on the mathematics of the future. It has not the simplicity and the inevitableness of the greatest work; it would be greater if it were less strange. One gift it shows which no one will deny—profound and invincible originality."

The Norwegian mathematician Atle Selberg, one of the great number theorists of this century wrote : "Ramanujan's recognition of the multiplicative properties of the coefficients of modular forms that we now refer to as cusp forms and his conjectures formulated in this connection and their later generalization, have come to play a more central role in the mathematics of today, serving as a kind of focus for the attention of quite a large group of the best mathematicians of our time.

Other discoveries like the mock-theta functions are only in the very early stages of being understood and no one can yet assess their real importance. So the final verdict is certainly not in, and it may not be in for a long time, but the estimates of Ramanujan's nature in mathematics certainly have been growing over the years. There is doubt no about that."Often people tend to speculate what Ramanujan would have achieved if he had not died or if his exceptional qualities were recognized at the very beginning. There are many instances of such untimely death of gifted persons, or rejection of gifted persons by the society or the rigid educational system. In mathematics we may cite the cases of **Niels Henrik Abel** (1809-29) and **Evarista Galois** (1811-32). Abel solved one of the great mathematical problems of his day - finding a general solution for a class equations called quintiles. Abel solved the problem by proving that such a solution was impossible. Galois pioneered the branch of modern mathematics known as group theory.

What is important is that we should recognize the greatness of such people and take inspiration from their work. Even after more than 80 years of the death of Ramanujan the situation is not very different as far the rigidity of the education system. Today also a 'Ramanujan' is not likely to get a chance to pursue his career. This situation remains very much similar as described by JBS Haldane (1982-1964), a British born geneticist and philosopher who spent last part of his life in India. Haldane said : "Today in India Ramanujan could not get even a lectureship in a rural college because he had no degree. Much less could he get a post through the Union Public Service Commission. This fact is a disgrace to India. I am aware that he was offered a chair in India after becoming a Fellow of the Royal Society. **But it is scandalous that India's great men should have to wait for foreign recognition**. If Ramanujan's work had been recognised in India as early it was in England, he might never have emigrated and might be alive today. We can cast the blame for Ramanujan's non-recognition on the British Raj. We cannot do so when similar cases occur today.

Ramanujan's brief life and death are symbolic of conditions in India. Of our millions how few get any education at all; how many live on the verge of starvation.

…………. Jawaharlal Nehru in his Discovery of India

This statement is very much valid even today. And for these very reasons the story of Ramanujan should be told and retold to our younger people particularly to those who aspire to do something extraordinary but feel dejected under the prevailing circumstances. And in this connection it is worthwhile to remember what **Chandrasekhar** had to say: "I can recall the gladness I felt at the assurance that one brought up under circumstances similar to my own could have achieved what I could not grasp. The fact that Ramanujan's early years were spent in a scientifically sterile atmosphere, that his life in India was not without hardships that under circumstances that appeared to most Indians as nothing short of miraculous, he had gone to Cambridge, supported by eminent mathematicians, and had returned to India with very assurance that he would be considered, in time as one of the most original mathematicians of the century — these facts were enough, more than enough, for aspiring young Indian students to break their bands of intellectual confinement and perhaps soar the way what Ramanujan had. " As someone has written "Ramanujan did mathematics for its own sake, for thrill that he got in seeing and discovering unusual relationships between various

mathematical objects." Today Ramanujan's work has some applications in particle physics or in the calculation of pi up to a very large number of decimal places. His work on Rieman's Zeta Function has been applied to the pyrometry, the investigations of the temperature of furnaces. His work on the Partition Numbers resulted in two applications — new fuels and fabrics like nylons. But then highlighting the importance of the application side Ramanujan's work is really not very important.

It is on April 26, 1920 the the Amaranthine flower was out of the branch of the maths sapling leaving its fragrance for ever to us. Alas! Ramanujan died of tuberculosis in Kumbakonam. He was only 32 years old. "It was always maths ... Four days before he died he was scribbling," said Janaki, his wife. The untimely death of Ramanujan was most unfortunate particularly so when we take into account the circumstances under which he died. As Times Magazine rightly wrote: "There is something peculiarly sad in the spectacle of genius dying young, dying with the first sweets of recognition and success tasted, but before the full recognition of powers that lie within." The only **Ramanujan Museum** in the country, founded by Shri P. K. Srinivasan, a mathematics teacher, operates from March 1993 in the Avvai Academy, Royapuram, Madras. The achievement of Ramanujan was so great that those who can really grasp the work of Ramanujan 'may doubt that so prodigious a feat had ever been accomplished in the history of thought'. I will stop here with the quotation by G.H. Hardy: "**Archimedes** will be remembered when **Aeschylus** is forgotten, because languages die and mathematical ideas do not. Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean."

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H.H. Swami Bharati Krishna Tirthaji

VEDIC MATHEMATICS : HIS GIFT TO THE WORLD

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The richness of cultural tradition of a country or a race leans heavily upon the development of science and technology in the country and this development depends upon the treasure of mathematical knowledge and its application in various spheres of public and personal life.

When we think about the contribution of India towards the world particularly regarding mathematics we notice that the natural numbers which the world know today are the boon of the Indian sages. The striking fact to note here is that the Hindu genius gave its perfection at the very beginning.

This heritage was maintained or rather given thrust by H.H. Swami Bhartikrishna Thirtha Ji. The purpose of this paper writing is to give a brief life sketch of Swamiji and his one of the gifts to the world known as 'Vedic Mathematics'. Certainly it is impossible for an ordinary person like me to say something about Swamiji in few words or sentences even though I am doing this job hesitatingly.

Swami Bhartikrishna Tirtha Ji was the Shankaracharya of the Govardhan Math, Jaganath Puri as well as Dwarka Gujrat. Swami Ji was born in 1884 and shed his mortal frame on 2nd Feb. 1960. His family background was very rich. His father P. Narsimha Shastri was then in service as a Tehsildar at Tinnivelly (Madras). His uncle Shri Chandra Shekhar Shastri was principal of Maharaja's College Vizianagaram, grandfather C. Ranganathan Shastri was Justice of Madrass High Court.

Swamiji named as Venkatraman in his early days was an exceptionally brilliant student and proved the same by excelling in every subject/Stream throughout his educational career. He passed his matriculation examination from the Madras University in January 1899 and topped the list with vibrant colors. He was extraordinary proficient in Sanskrit and oratory and on account of this he was awarded with the title 'Saraswati' by the Madras Sanskrit Association in July, 1899. At the young age of twenty one he passed M.A. in seven subjects including Science, Mathematics, English, History, Philosophy simultaneous by securing the highest honours in all.

When he was studying in the university. he used to write articles related to philosophy, sociology, religion etc. He was much interested in the latest researches and development in modern science, tempermanently he was interested in Adhyatma Vidya. This interest led him to Sringeri Math in Mysore in 1908 to lay himself at the feet of the renowned late Jagadguru Shankaracharya Sri Satcidananda Sivabhinava Narsimha Bharti Swami. After devoting sufficient time there he assumed the post of principal of National college. For him this assignment was a national duty. But his natural spiritual instinct drove him again to Swami Satcidananda at Shringeri in 1911. The next 8 year he spent in the profound study of the most advanced Vedanta philosophy and practice of Brahma Sadhana. The scholars working in the field used to invite him to deliver lectures at different places like Amelner, Pune , Bombay.

After most advanced studies, the deepest mediation and the highest-spiritual attainments Prof. Venkatraman Saraswati was initiated into the holy order of Samnyas at Varansi (Banaras) by the holiness Jagadguru Shankaracharya Sri Trivikram Tirth Ji Maharaj of Sharda Peeth on 4th July 1919. This was also the occasion when he was given the new name "Swami Bharti Krishna Tirtha".

He prove by his performance for being installed on the pontifical throne of Shardha Peeth Shankaracharya. Despite his reluctance and active resistance he was installed as Jagadguru of Shardapeeth with formal ceremonies.

Jagadguru of Govardhan Peeth was also highly impressed by Swami ji. Due to ill health he requested Swami ji to accept the Gadi of Govarhdan Math. Swami ji couldn't afford to deny. In the capacity of Jagad guru he continued the spiritual teachings of Sanatan Dharma for almost 4 decades. He was against escapism from duties under the grab of spirituality. His great emphasis was on the necessity of harmonizing the spiritual and the material spheres of daily life.

He was of the view that the progress must be simultaneously of both the individual and society towards speedy realization of India's spiritual and cultural ideals.With these ideas in mind he tried to develop a system which helps in total construction first of India and through it the world.For this he founded Vishwa Punarnirman Sangha (World Reconstruction Association), many prominent figures of the society came together to work for it.But due to ill health the Sangh couldn't work effectively.

For the cause of world peace and to spread the spiritual ideals even out side India Swami ji, for the first time ,went on tour to America in Feb 1958. The tour was sponsored by self realization fellowship of Los Angeles he was invited to give talks and mathematical demonstration on the television . Swamiji also gave some lectures in U.K. during this tour. He came back to India in may 1958.

He was having a winning personality, charming innocence, avid thirst for knowledge religious zeal he never cared for position. People from all walks of life attracted to him due to Bhakti, his simplicity, impartial behaviour A sharp intellect, a retentive memory and keen zest went to mark him as the most distinguished scholar of the time. He was also a poet and write a number of poems in Sanskrit.He was quite active throughout his life. After coming from the tour of U.S.A. in 1958 his health deteriorated continuously and he took Samadhi in 1960.

Swami ji Discovered Vedic Mathematics

Swamiji, who was an accomplished Vedic scholar wrote 16 volumes on Vedic mathematics, comprehensively in all branches of mathematics. But it is learnt that all the volumes were lost mysteriously. Despite his poor health and weak eyesight Swamiji with his untiring capacity, strong will and determination wrote a comprehensive book on Vedic mathematics.

The terms Vedic mathematics refer to a set of 16 sutras and 13 upsutras. This speaks for its coherence and simplicity in handling mathematical problems. The sutras not only develop aptitude and ability but also nurture and develop logical thinking and intelligence and encourage innovativeness. While doing arithmatical calculation we deal with numbers 0 to 9. Each of these has got peculiar properties and ways of behavior. If they are closely observed, we can have 'novum' methods of doing calculation.

Vedic mathematics has got both these advantages. It is a good means of performing calculation mentally, it also keeps before us more than one way of doing particular calculation. This thing is possible because Vedic maths looks at numbers from different angles. When 89 is looked at as 89 it has some properties. But as soon as we look at it as go 90-1 ($9\bar{1}$) we have altogether new world opened up for calculation. One more feature of it is that most of the time it looks for patterns of numbers instead of individual digits. Instead of treating 997 as any three digit number it looks at the whole number and recognizes it as a number near a base. Once this property is recognized, easier methods of multiplication, division etc. can be adopted.

Also in this approach we find that in Vedic Mathematics instead of emphasing on individual digit, attention is laid on interrelation of digits, multiplication and recurring decimals are examples of this feature.

Also it is equally important to observe that after sufficient practice this develops into outlook which is becoming more and more desirable these days. Our fragmented approach towards everything has created all sorts of problems. Through Vedic maths we can develop a holistic approach we can start looking at the interrelatedness of everything it will make our life better.

Today particle physics tells us about the interconnectedness of everything in the universe. Vedic maths does the same in its own way. This philosophical quality of Vedic maths will help us to develop a holistic ecological approach towards life. Because of these qualities Vedic maths has certain advantages. First the calculation become easier. This can be done with less mental energy and consuming less time, but at the same time there is full involvement. This is very important calculators to give answers in limited time and with still engaging less mental energy but our involvement in the process of calculation is nil.This is highly dangerous in the long run as we lose our ability to calculate. Vedic maths on the contrary helps us to sharpen our calculating ability, secondly, because of the simplicity and availability of more than one method, the job of calculating becomes easy and interesting.

In this way if we emphasize on one thing that Vedic maths is not just some tricks of doing mathematical calculations, it goes far beyond this and becomes science in itself, beneficial in solving problems related to higher mathematics i.e. solution of simultaneous linear equations evaluation of determinants, transcendental equations, solution

of cubic and higher order equations, solution of linear and non linear differential equation and partial differential equation etc, putting Vedic maths as a science, and not as a pack of tricks and magic this become important. There is a vast scope of research in the subject and with the co-ordinated efforts of scholars in Vedic mathematics, Sanskrit and Computer technology, a lot can be achieved.

The objective of human personality is the realization of true knowledge, in other words it can be said that true knowledge can be realized if the difference between the objective component of knowledge generated by intellectual component and subjective component of knowledge generated by the emotive component of the individual mind is eliminated.

It may be mentioned that the ancient Indian system of thought is a comprehensive system of the realization of true knowledge. so as to attain a state of non-variable happiness.

The creations of Swamiji has the same spirit – Vedic mathematics given by Swamiji not only solve the problem related to mathematics but also develop the personality by including all possible dimensions. According to anatomist the left half of the brain systematically collects information does sequential analysis and arrives at logical conclusions and results. Only the left half of the brain is activated and developed by the present maths educations being given in the schools and colleges. The right half of the human brain works with pattern recognition and has the capability of intuition. This most powerful facet of human personality the intuitive faculty, unfortunately, remains undeveloped in most of the students. In the Vedic mathematics system, the very first step is to recognize the pattern of the problem and pick up the most effective Vedic algorithm. Further at each subsequent step we recognize the pattern and complete the task by using the appropriate superfast mental working procedure, because in Vedic maths we have multiple choice available at each stage of working. This practice systematically develops the right half of the brain as well. Thus Vedic maths activates the brain.

Swami ji was visionary and his creation also have the same potential. Following his teaching one can attain the same hights as visualized by him. Seeking the benediction of Swami Bharti Krishna Tirth, let us come together and put a step forward towards the new and clear vision of life for the betterment of society.

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